Macroscopic avalanches in motion by curvature with many obstacles

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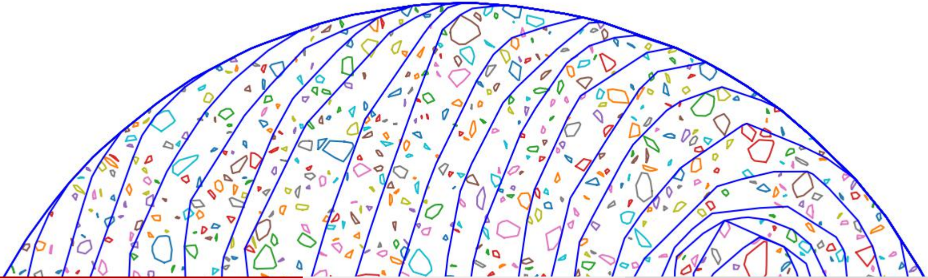

1. Graphene de-adhesion / Imbibition
2. Convexification and Merging model
3. Analysis of the transition
4. Link to boostrap percolation models
5. Very low initial disc densities
6. Cluster size distribution
7. Graphene de-adhesion by intercalated particles
8. Conclusion

Convexification

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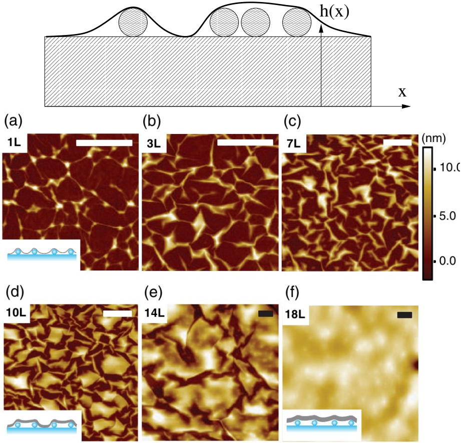
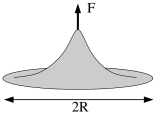
Few-layer graphene de-adhesion caused by intercalated nanoparticles



M. Yamamoto, OPL, J Huang, WG Cullen, TL Einstein, MS Fuhrer, Phys Rev X 2012

Few-layer graphene on Nanoparticles: collective effects



Föppl Von Karman equations

Schwerin (1929)

diameter detach. zone:
$2r_d = d(4nG/3\gamma_n)^{1/4}$

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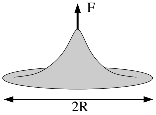

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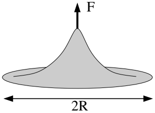

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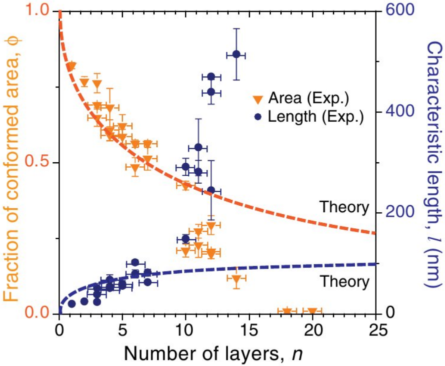

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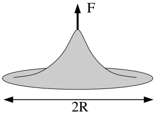

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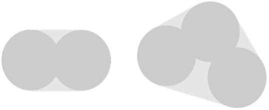
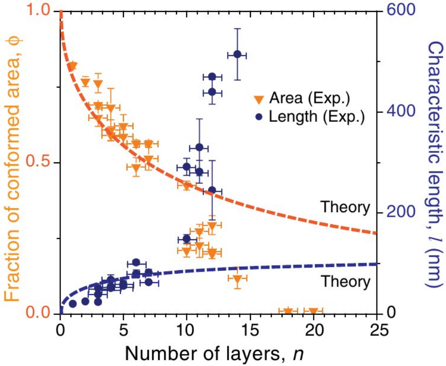
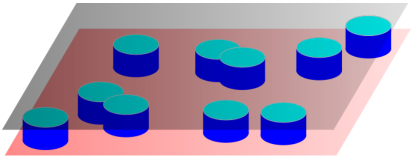




Concave parts detach from the substrate?

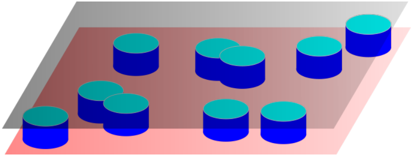
→ Collective effects ?

Imbibition with pinned wet regions



- Hele-Shaw cell with wet zones
- Liquid reservoir with zero-pressure
- Initial state with disc-shape wet regions

Imbibition with pinned wet regions



A simple model:
Surface tension → motion by curvature

$$v_n = -\kappa$$

- Hele-Shaw cell with wet zones
- Liquid reservoir with zero-pressure
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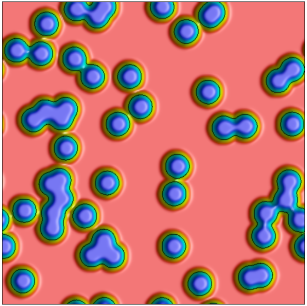
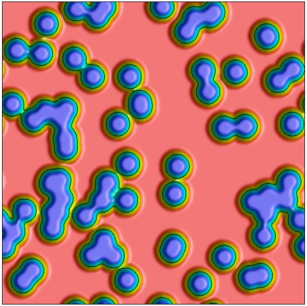
Imbibition with pinned wet regions

Interface motion by curvature + pinned positive-curvature disc edges
Motion by *negative* curvature $v_n = -\min[\kappa, 0]$

50 discs



70 discs



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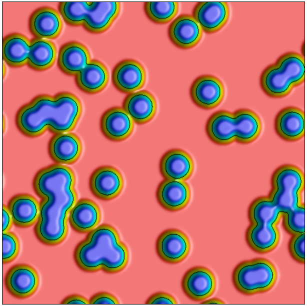

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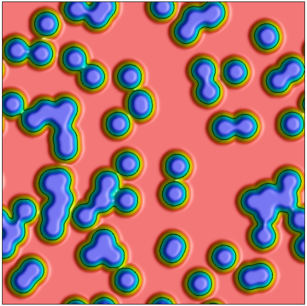

Imbibition transition!

Convexification

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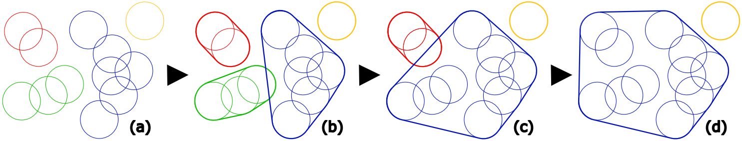
The model

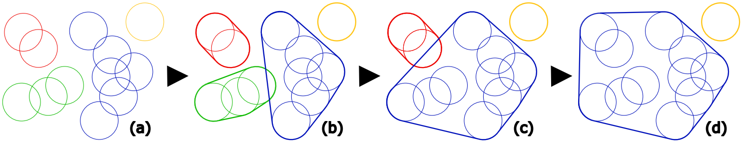


A simple model

1. Start with an initial condition (initial domains = randomly scattered discs)

Our study focuses on the final state

The model



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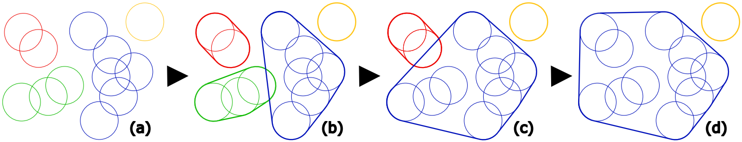

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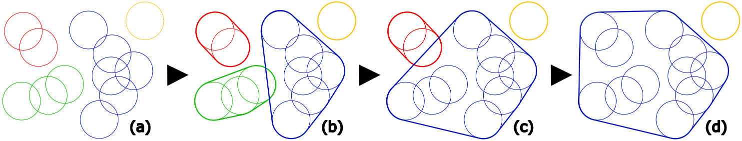

A simple model

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3. Take the convex hull of a each domain
4. Return to 2

Our study focuses on the final state

Adding discs one by one

Parameters

- Disc radius r_d (fixed)

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Two dimensionless numbers

- Concentration

$$C = \pi r_d^2 \rho_d = \frac{N_d \pi r_d^2}{A_{syst}}$$

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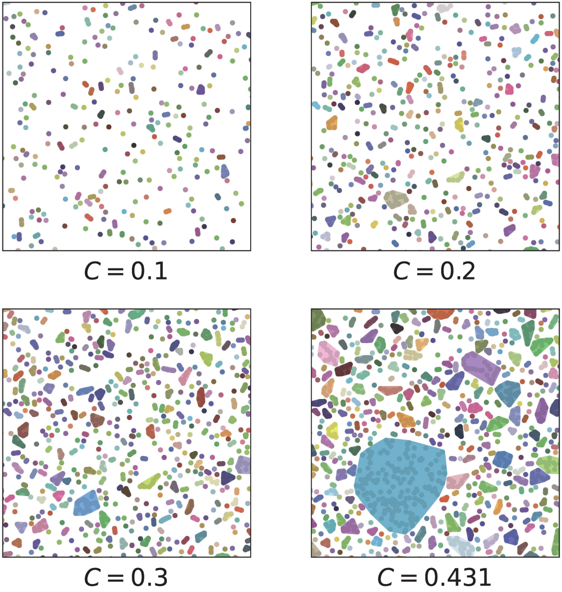
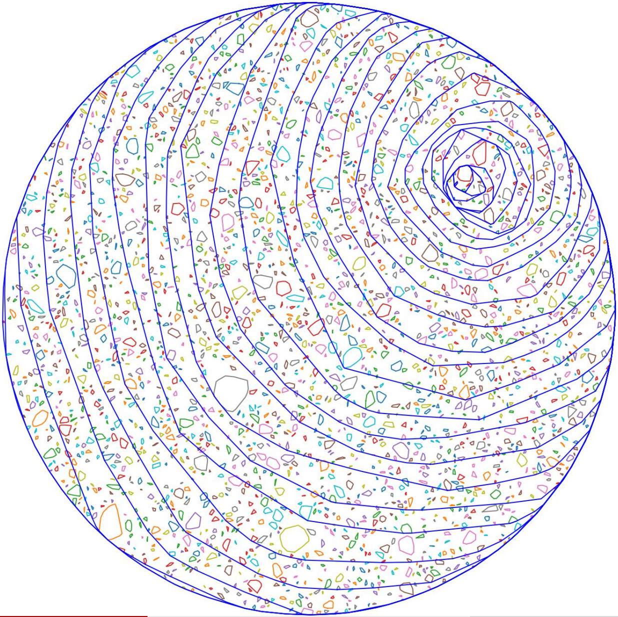


Macroscopic avalanche adding one disc (43122th disc with $\bar{A}_{syst} = 10^5$)



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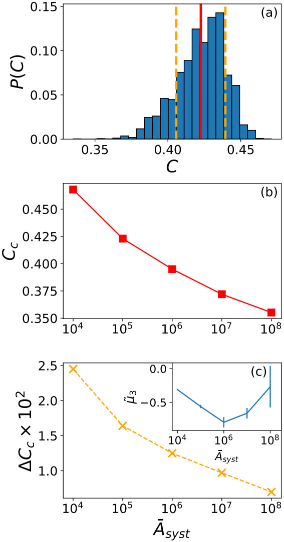
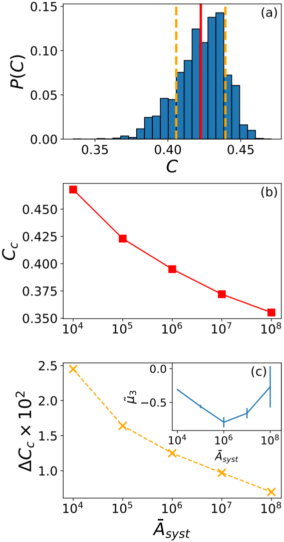
Transition



0.15
0.10
0.05
0.00
P(C)
(a)
0.35
0.40
0.45
C
(b)
0.450
0.425
0.400
0.375
0.350
C_c
10^4
10^5
10^6
10^7
10^8
2.5
2.0
1.5
1.0
$\Delta C_c \times 10^2$
(c)
0.0
−0.5
$\tilde{\mu}_3$
$\bar{A}_{syst}$

Transition



- Transition threshold C_c decreases with $\bar{A}_{syst}$

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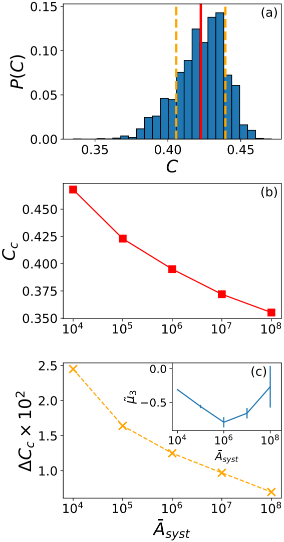
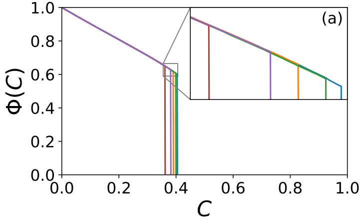


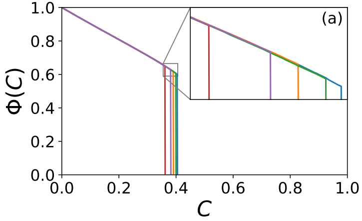
- Transition threshold C_c decreases with $\bar{A}_{syst}$
- Transition width ΔC_c decreases with $\bar{A}_{syst}$

$\langle \Phi \rangle$ as an order parameter



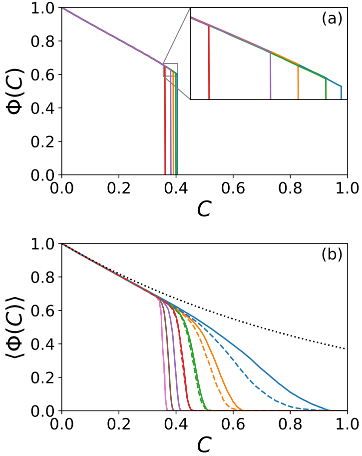
- $\Phi(C)$ non-invaded fraction

$\langle \Phi \rangle$ as an order parameter



- $\Phi(C)$ non-invaded fraction
- Discontinuous transition for each realization
- Spreading of transition thresholds

$\langle\Phi\rangle$ as an order parameter



$\bar{A}_{syst} < 10^8$, $\geq 10^3$ realiz.; $\bar{A}_{syst} = 10^8$, 60 realiz.

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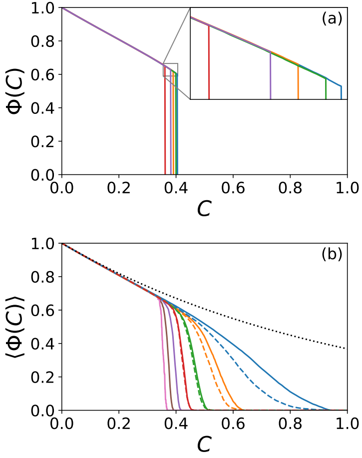

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- $\langle \Phi \rangle$ averaged over realizations
- Similar transition for fixed or periodic boundary conditions

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Bootstrap percolation

On-lattice boostrap percolation on square lattice

1. Remove randomly particles at N_i sites

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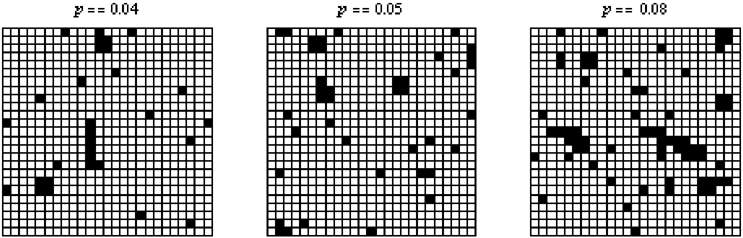

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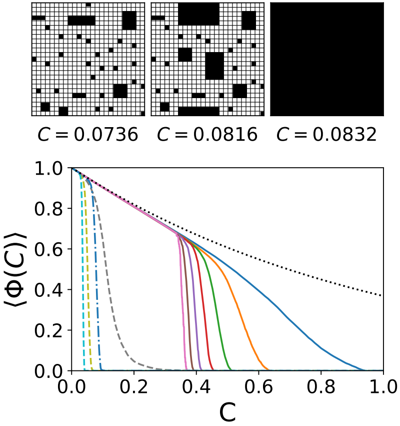

BP: 10^2; 10^4; 10^6

Correspondence

- $C = N_i / N_{latt}$

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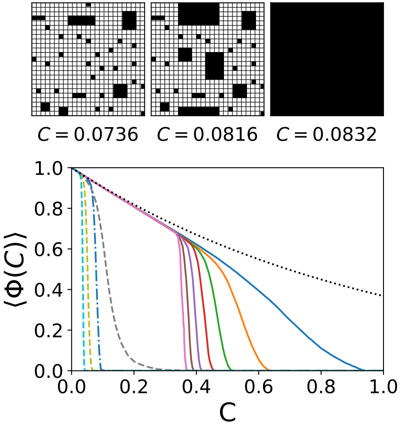

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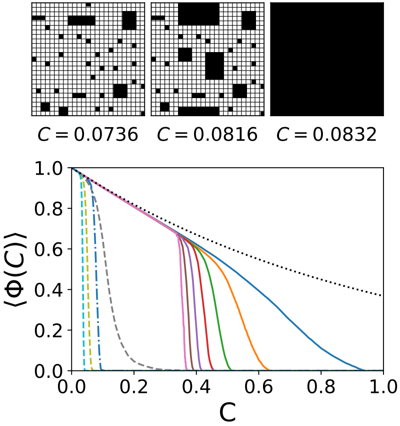

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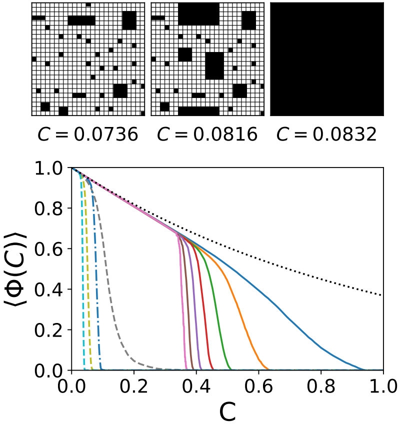

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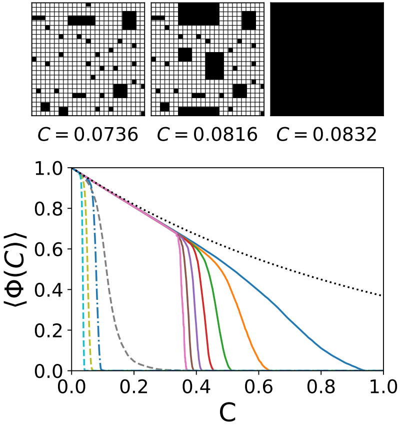

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- In BP simulations $C_c \xrightarrow[\bar{A}_{syst} \to +\infty]{} ??$

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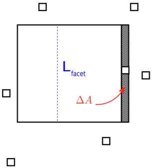

Merging with one site

$$\Delta A = L_{facet} \sim A^{1/2}$$

Threshold

$$C_c \sim \frac{1}{\ln \bar{A}_{syst}} \xrightarrow[\bar{A}_{syst} \to +\infty]{} 0$$

Rigorous result A. E. Holroyd, 2003

$$\lim_{\substack{C \to 0 \\ A_{syst} \to \infty}} C \ln \bar{A}_{syst} = \frac{\pi^2}{9}$$

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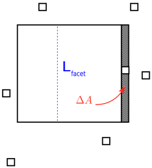

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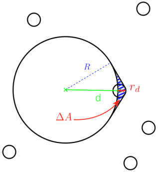
Convexification model



Merging with one disc

$$\Delta A \approx \frac{2^{3/2}}{3} R^{1/2} (1+\alpha)^{3/2} r_d^{3/2} \sim A^{1/4}$$

Threshold

$$C_c \sim \frac{1}{(\ln \bar{A}_{syst})^{2/3}} \xrightarrow[\bar{A}_{syst}\to+\infty]{} 0$$

No rigorous result!

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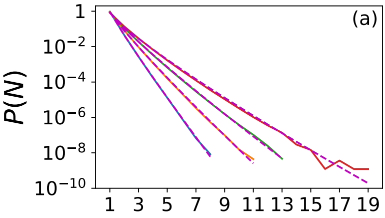

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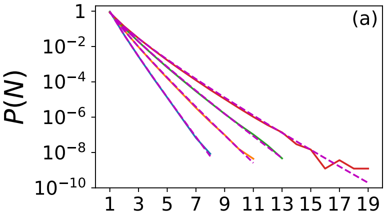

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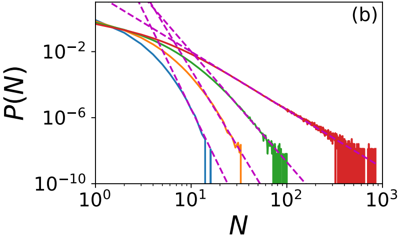
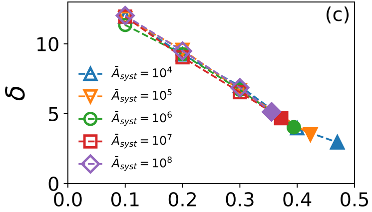
Simulations are much too small to check the asymptotics of the transition $C \rightarrow 0$!

Cluster size distribution: finite C

At finite $C \gtrsim 0.1$, power-law tails $P(N) \sim N^{-\delta}$



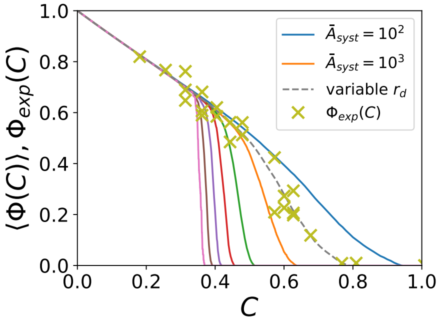


Linear dependence of the exponent: $\delta(C) \approx 14.19 - 24.99C$

Convexification

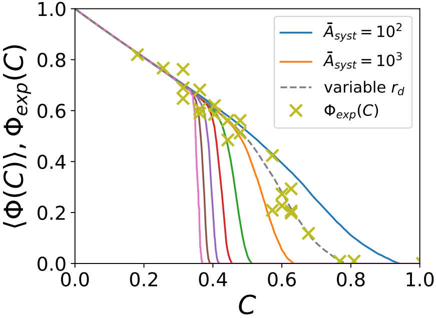
1. Graphene de-adhesion / Imbibition
2. Convexification and Merging model
3. Analysis of the transition
4. Link to boostrap percolation models
5. Very low initial disc densities
6. Cluster size distribution
7. Graphene de-adhesion by intercalated particles
8. Conclusion

variable r_d



- Simulations with variable r_d and with fixed N_d
  $\rightarrow$ both $C = N_d \pi r_d^2 / A_{syst}$ and $A_{syst} = A_{syst}/\pi r_d^2$ vary

variable r_d



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  $\rightarrow$ both $C = N_d \pi r_d^2 / A_{syst}$ and $A_{syst} = A_{syst}/\pi r_d^2$ vary
- Effective system size $N_d \approx 200$, or size 1.3μm

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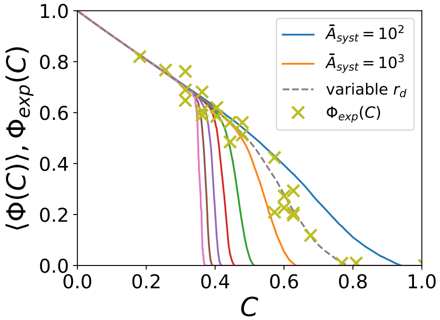

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- Effective system size $N_d \approx 200$, or size 1.3μm
- Interpretation: elasticity, correlations of particle positions ?

Convexification

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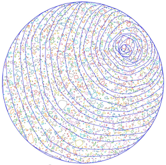
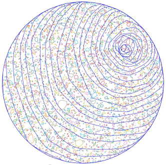
Conclusion



- A simple model: convexification + merging
- Applications: de-adhesion of graphene with intercalated particles, imbibition, monolayers on solid surface with impurities, clustering/classification with linear separability, etc.
- Many open questions: link to bootstrap percolation; large size asymptotics; origin of power-law tails at finite C –also in lattice boostrap percolation?

Conclusion



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