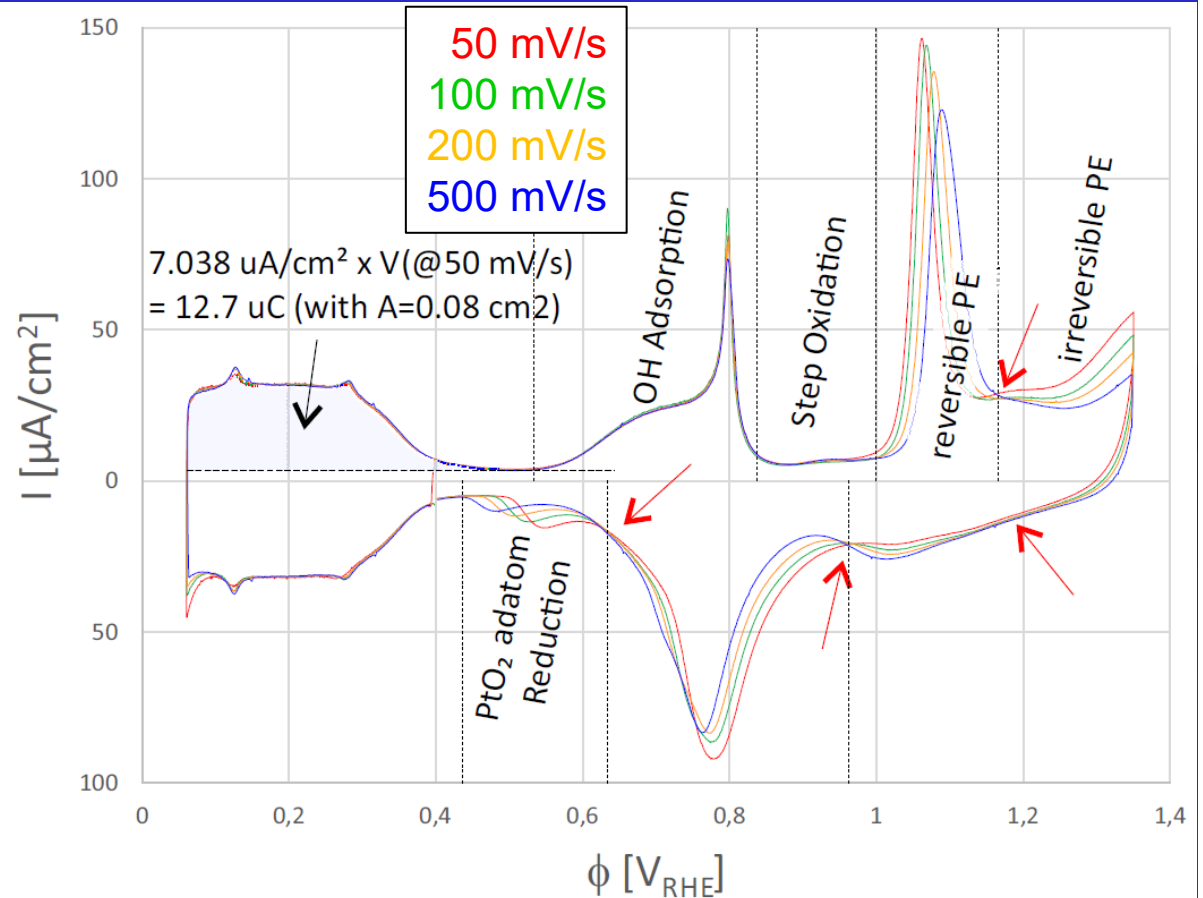
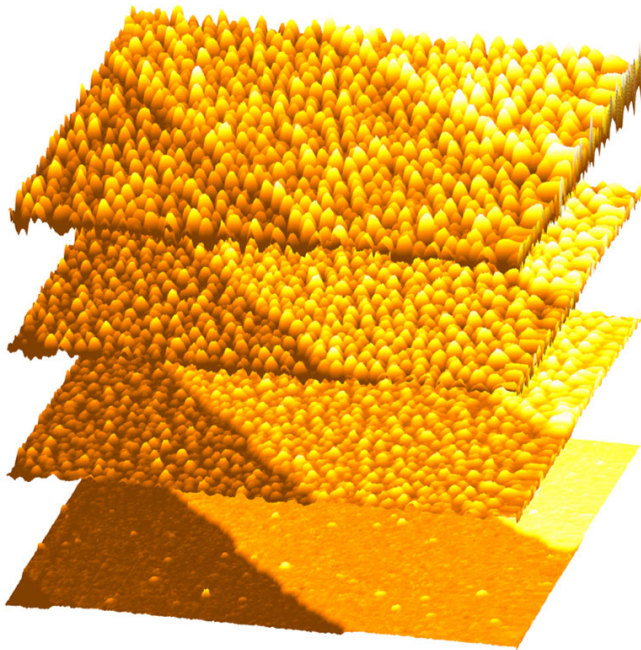
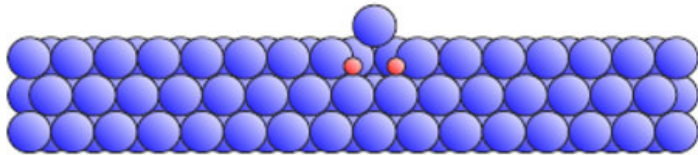


Arrhenius follows Frumkin

to describe Atomic Diffusion involved Peaks in Cyclic Voltammograms:
the Reversible Place-Exchange on Pt(111)

M.J. Rost

Kamerlingh Onnes Laboratory, Leiden University (NL)



Surprising Aspects of Pt(111) Oxidation and Reduction: Unravelling Four Different Oxidation Stages

EN-ROADS (MIT) <https://en-roads.climateinteractive.org>

EN-ROADS

English

Simulation

Graphs

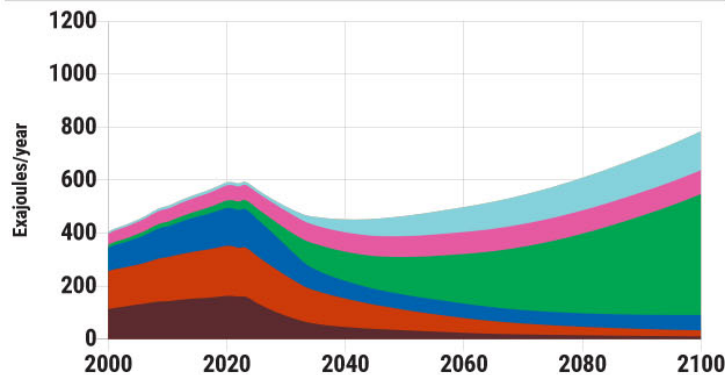
View

Help



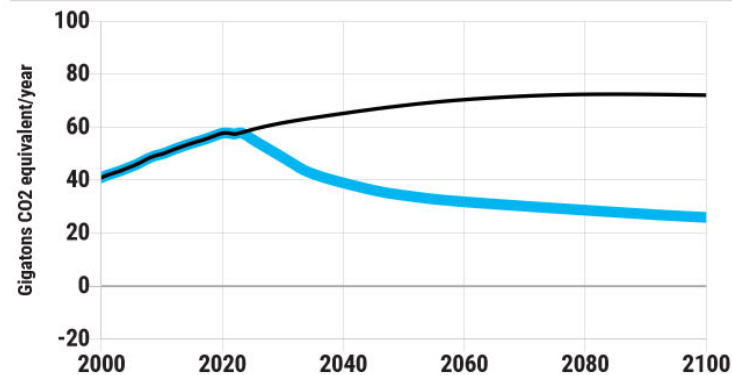
Share Your Scenario

Global Sources of Primary Energy



COAL OIL GAS RENEWABLES BIOENERGY NUCLEAR NEW ZERO

Greenhouse Gas Net Emissions

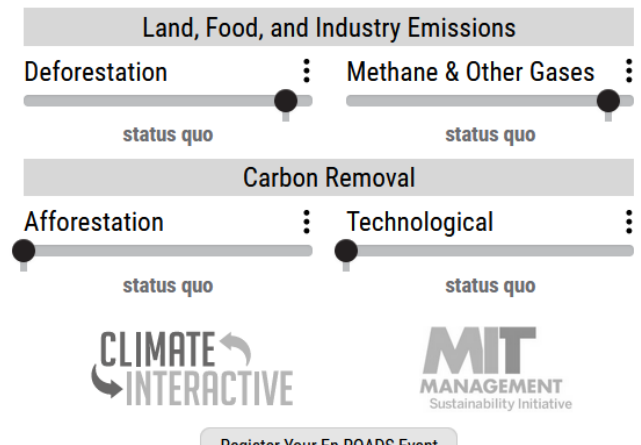
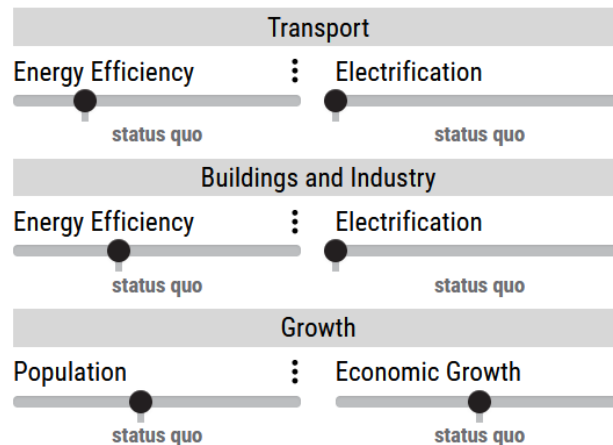
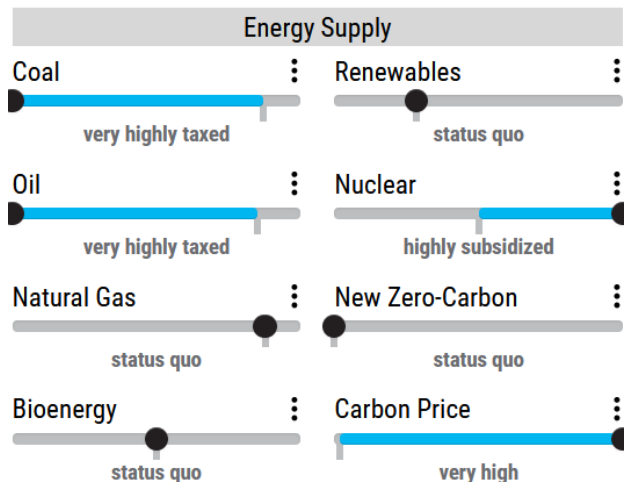


BASELINE CURRENT SCENARIO

+2.2°C

+4.0°F

Temperature Increase by 2100



CLIMATE INTERACTIVE

MIT MANAGEMENT Sustainability Initiative

Register Your En-ROADS Event

v23.9

Rost@physics.leidenuniv.nl

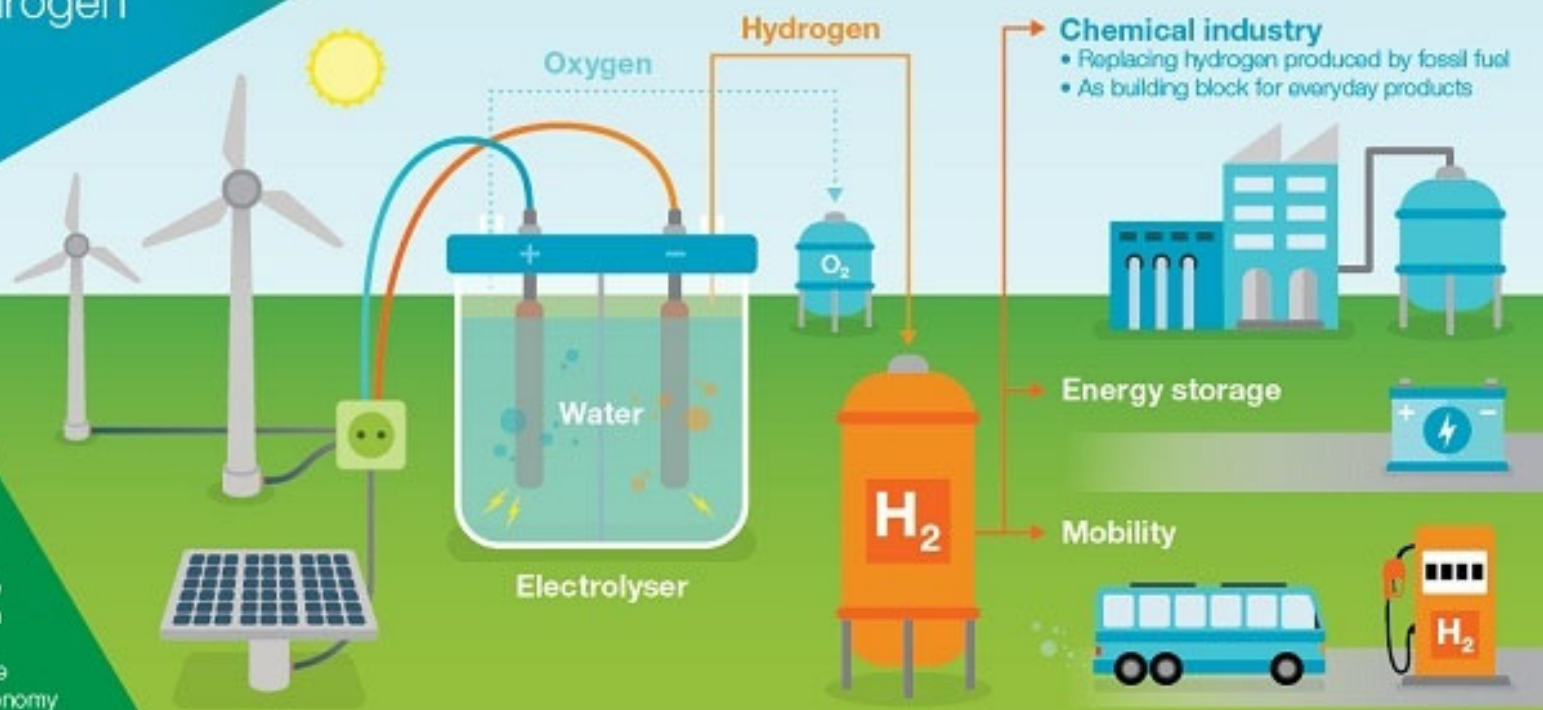
www.physics.leidenuniv.nl/rost

Surprising Aspects of Pt(111) Oxidation and Reduction: Unravelling Four Different Oxidation Stages

AkzoNobel and Gasunie
jointly scaling up the conversion
of sustainable electricity
into green hydrogen

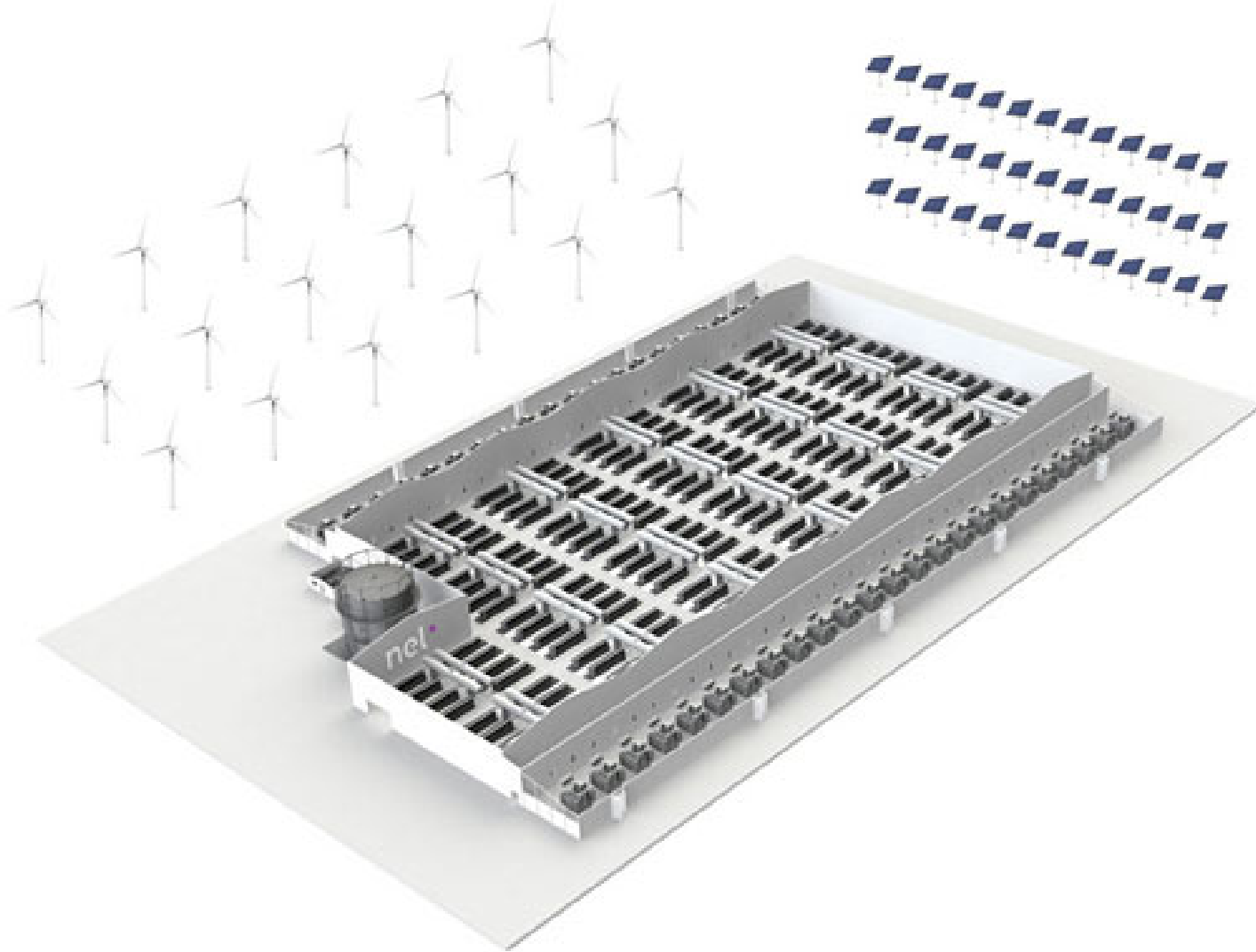
AkzoNobel
SPECIALTY CHEMICALS

- 20 megawatt water electrolyser, the largest in Europe
- Producing 3 kiloton (30 million m³) of green hydrogen
- Enough to fuel 300 buses, or enough for 1 bus to drive around the world more than 1100 times
- Upscaling is essential for the transition to a hydrogen economy





Background / Problem:



Rost@physics.leidenuniv.nl
www.physics.leidenuniv.nl/rost

The Future: Fuel Cells

T E S L A

Model S Model 3 Model X Model Y Charging



Jeff Dahn

@ Dalhousie University

gross weight 2265 kg
payload 439 kg

Model S

Plaid



J. Electrochem. Soc. **166** A3031 (2019)

Abstract:

...We conclude that cells of this type should be able to power an electric vehicle for **over 1.6 million kilometers** (1 million miles) and last at least **two decades in grid energy storage**. ...

600 km

Range
(WLTP)

2.1 s

0-100 km/h*

322 km/h

Top Speed*

1,020 hp

Vehicle Power†

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www.physics.leidenuniv.nl/rost

The Future: Fuel Cells

MB GenH2 Truck specs:

up to 1,000 km

80 kg liquid H₂

300 kW fuel cell

70 kWh battery

2 electric motors with 2x 230 kW

2x 1,577 Nm of torque

gross weight 40 tons

payload 25 tons



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www.physics.leidenuniv.nl/rost

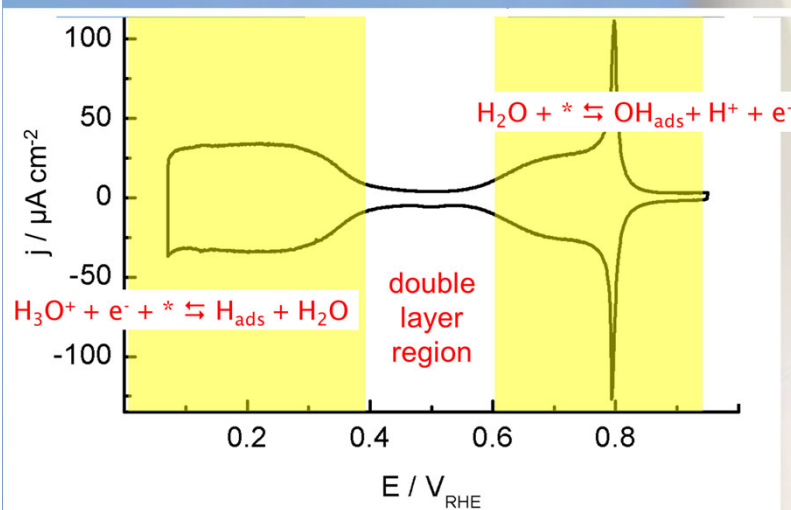
The Future: Fuel Cells



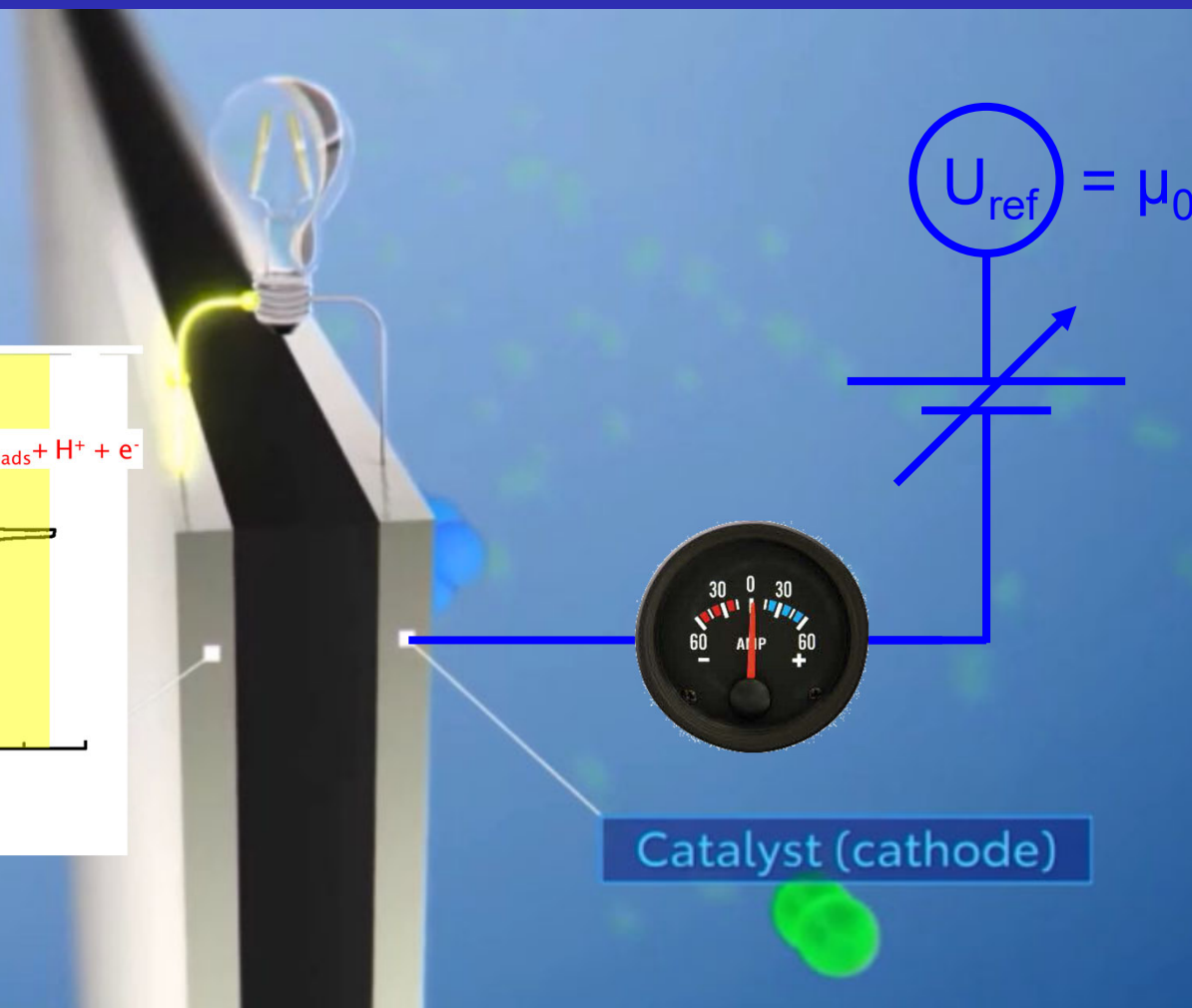
YouTube: 2021 Toyota Mirai - how it works and what's changed

The Future: Fuel Cells

How it works

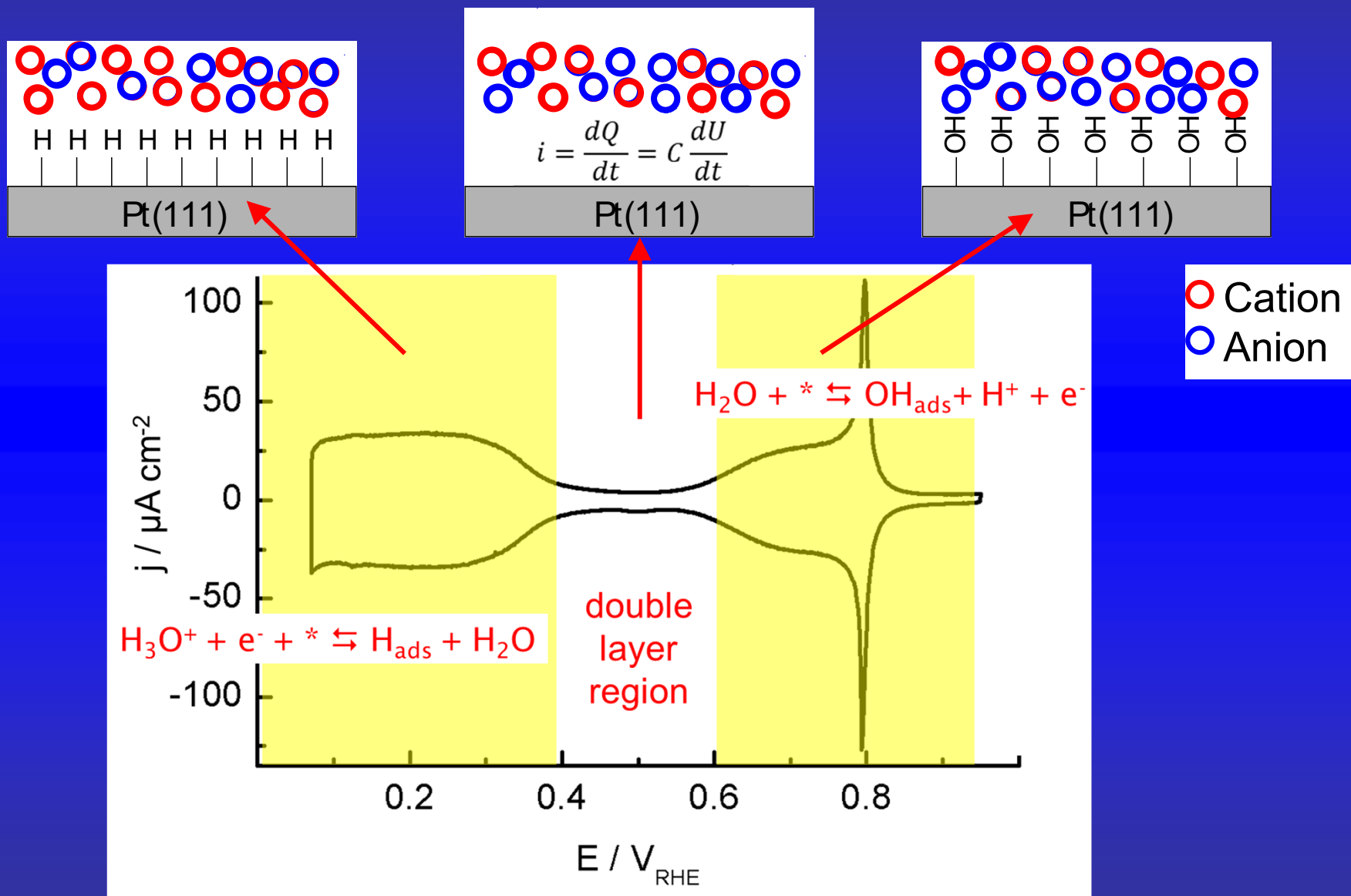


Catalyst (anode)



YouTube: 2021 Toyota Mirai - how it works and what's changed

Pt(111) Electrochemistry in 0.1 M HClO₄:



Gómez-Marín, A., & Feliu, J. *Electrochim. Acta*, **2012**, 82, 558–569



ON

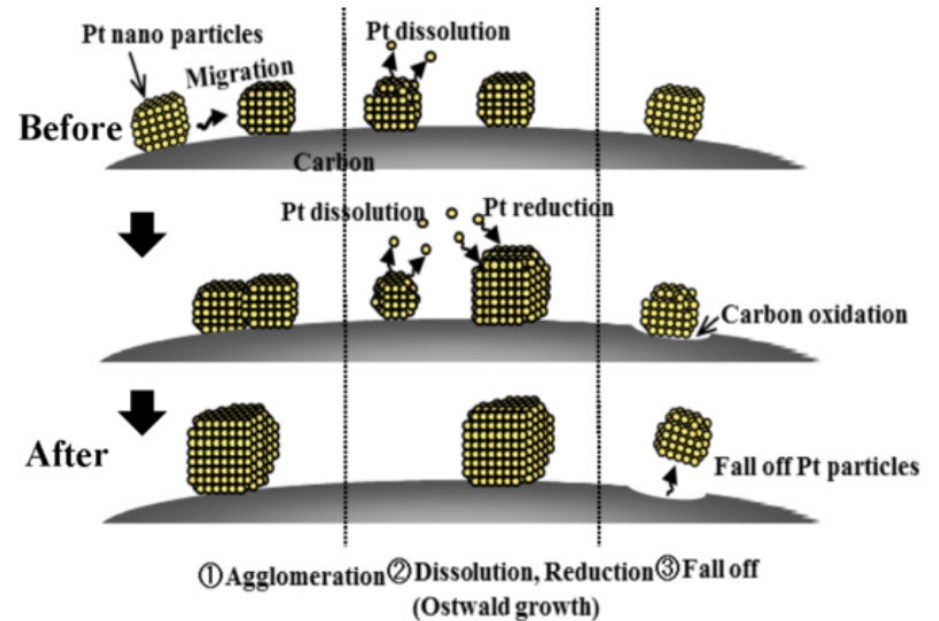
Background / Problem:

Green Car Congress

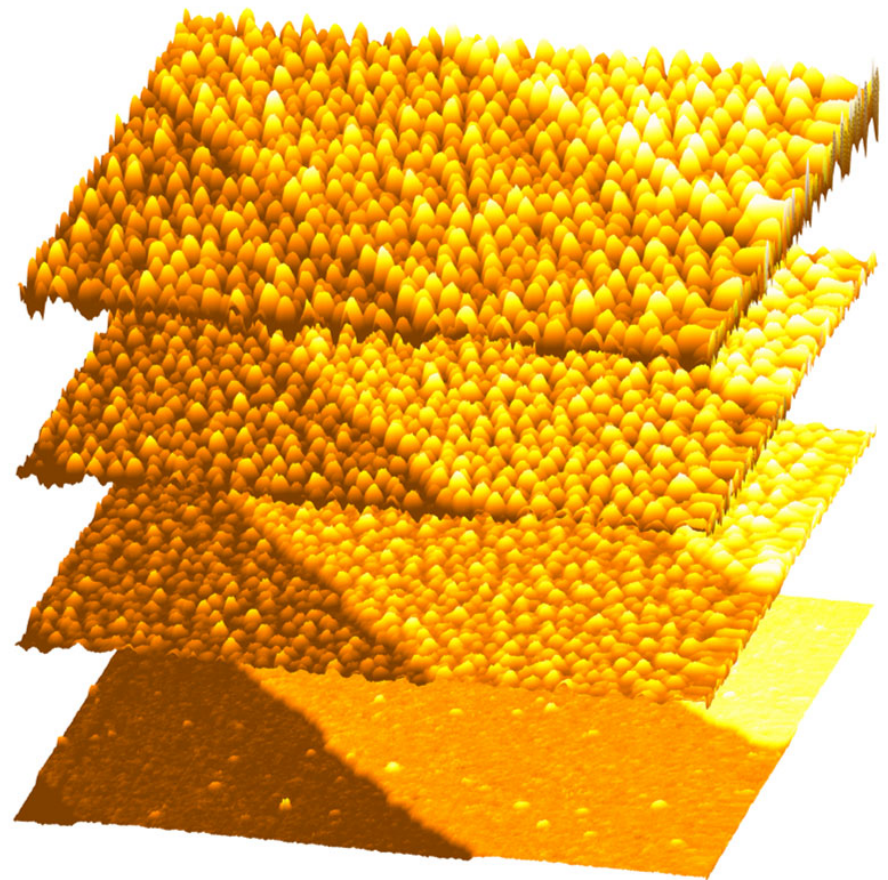
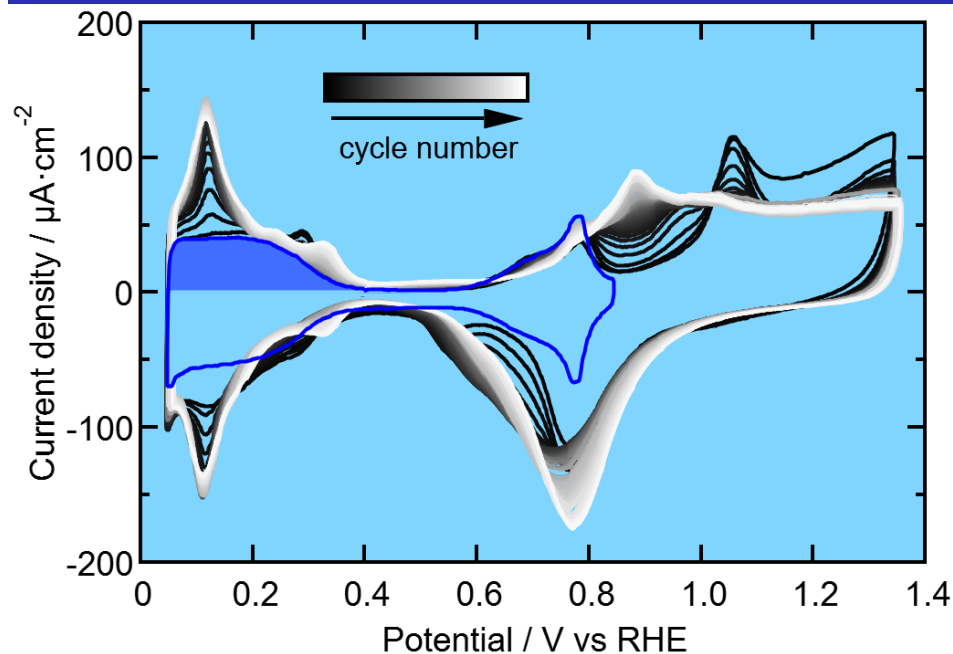
Energy, technologies, issues and policies for sustainable mobility

Toyota details design of fuel cell system in Mirai; work on electrode catalysts

While other major automakers have either introduced (Hyundai, Honda) or are in serious development of new hydrogen fuel cell vehicles for the market, Toyota continues to take the point in not just promoting, but also supporting the broader technical (and infrastructure) development required for a large-scale realization of hydrogen-based electromobility.



Atomic-Scale Identification of the Electrochemical Roughening of Platinum:



Surface & Electrochemical Evolution:

Correlation of Surface Site Formation to Nanoisland Growth
in the Electrochemical Roughening of Pt(111)

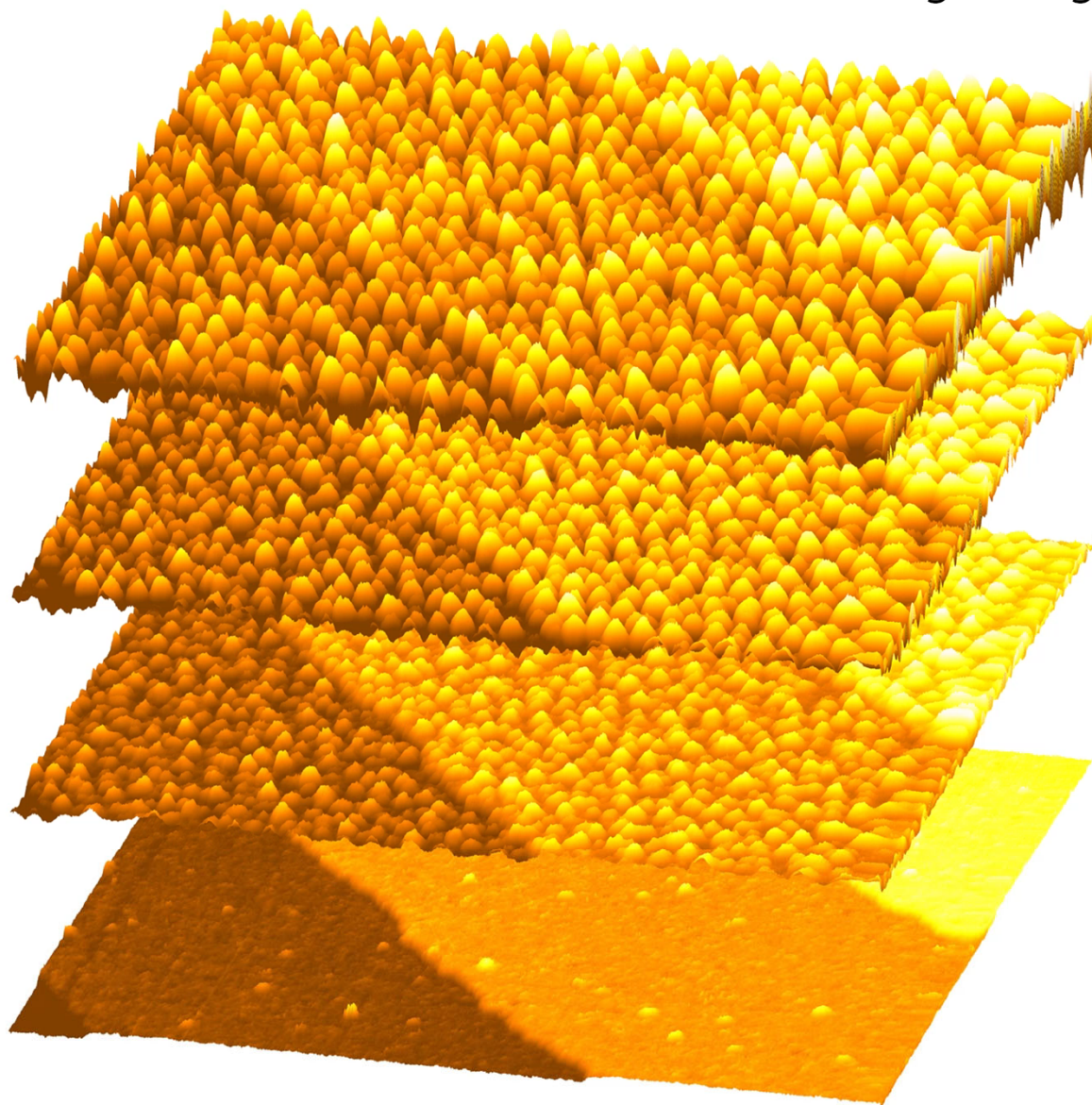
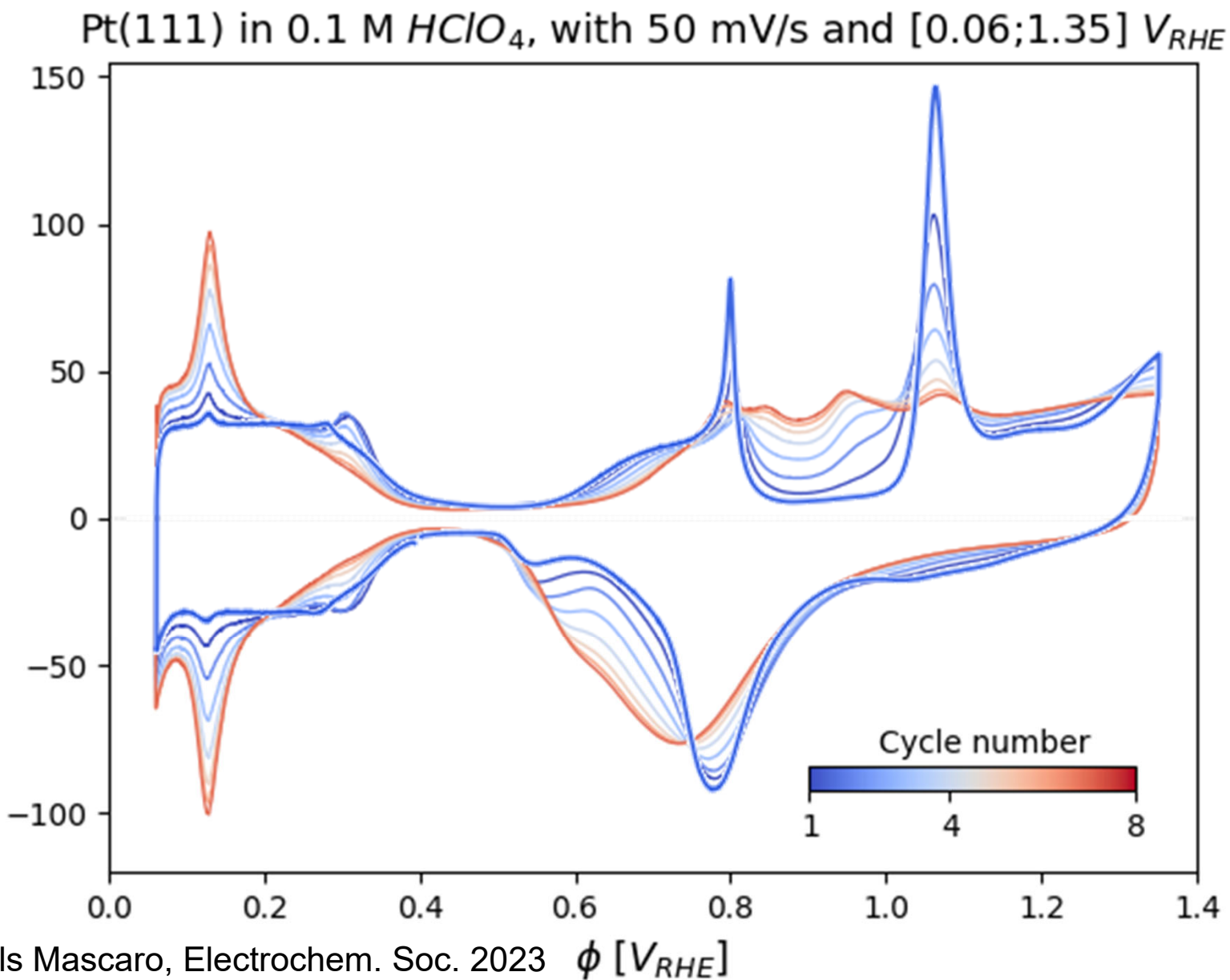


Image size: 230 x 230 nm²
Measured in the double layer region:
 $U_s = 0.4$ V and $U_t = 0.45$ V (vs. RHE)
after one or more CVs to 1.35 V

Roughening Process:

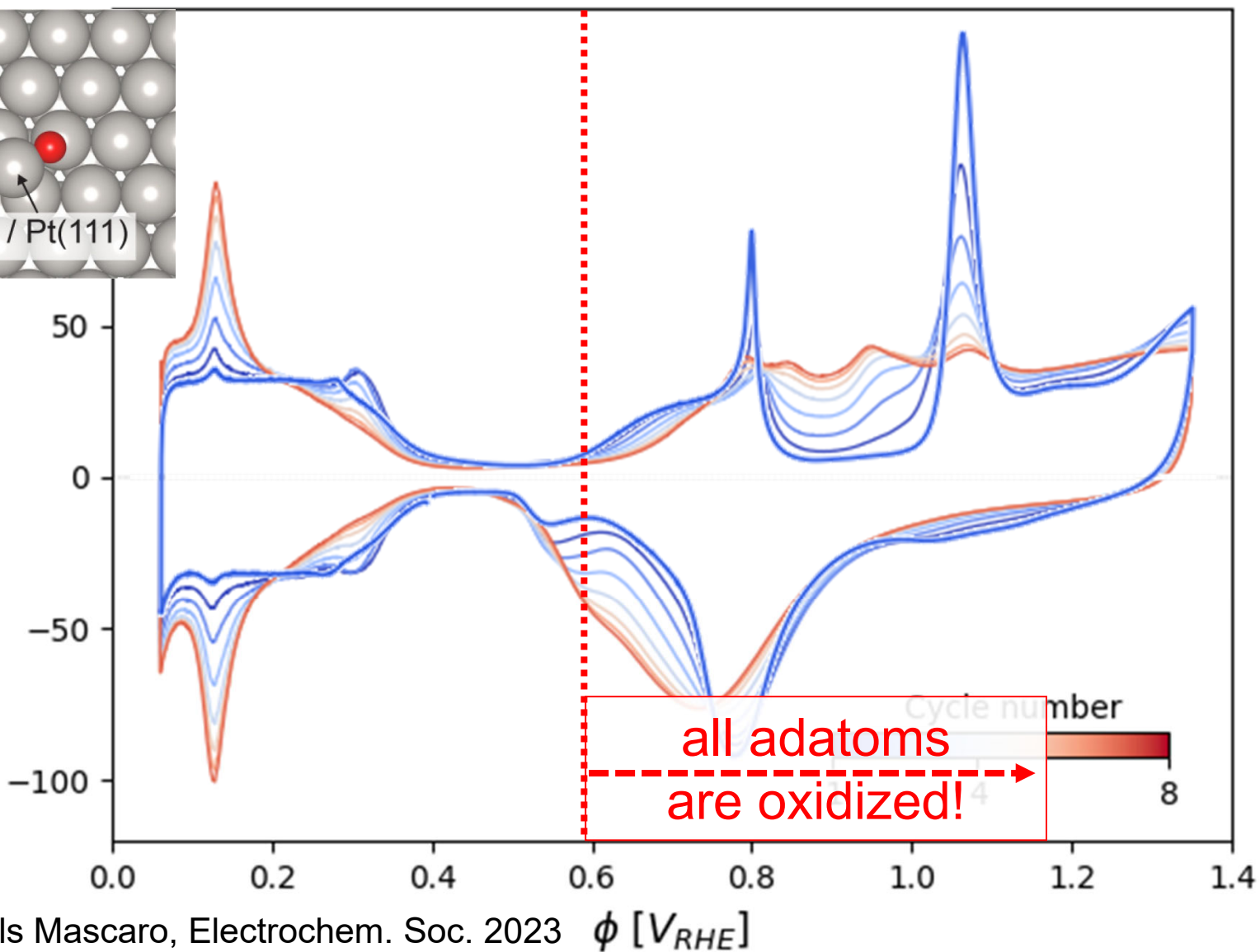
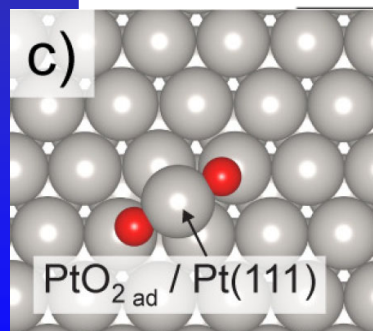


All the Oxides...



All the Oxides...

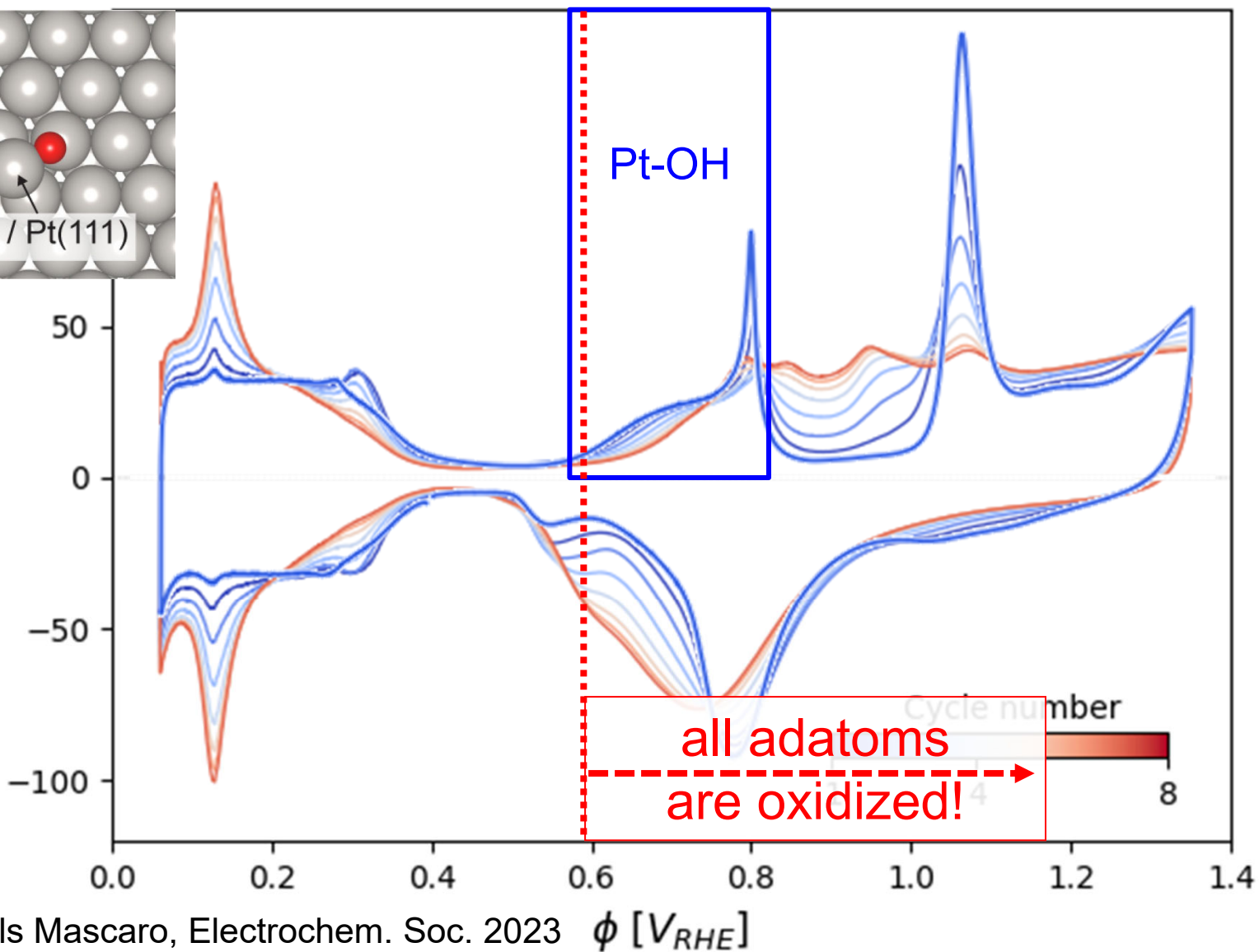
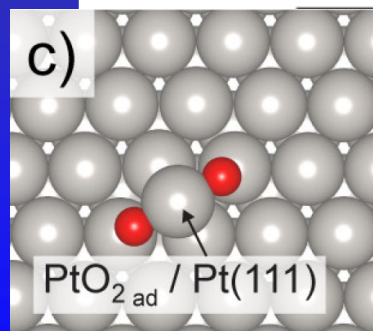
Pt(111) in 0.1 M $HClO_4$, with 50 mV/s and $[0.06; 1.35] V_{RHE}$



Valls Mascaro, Electrochem. Soc. 2023 $\phi [V_{RHE}]$

All the Oxides...

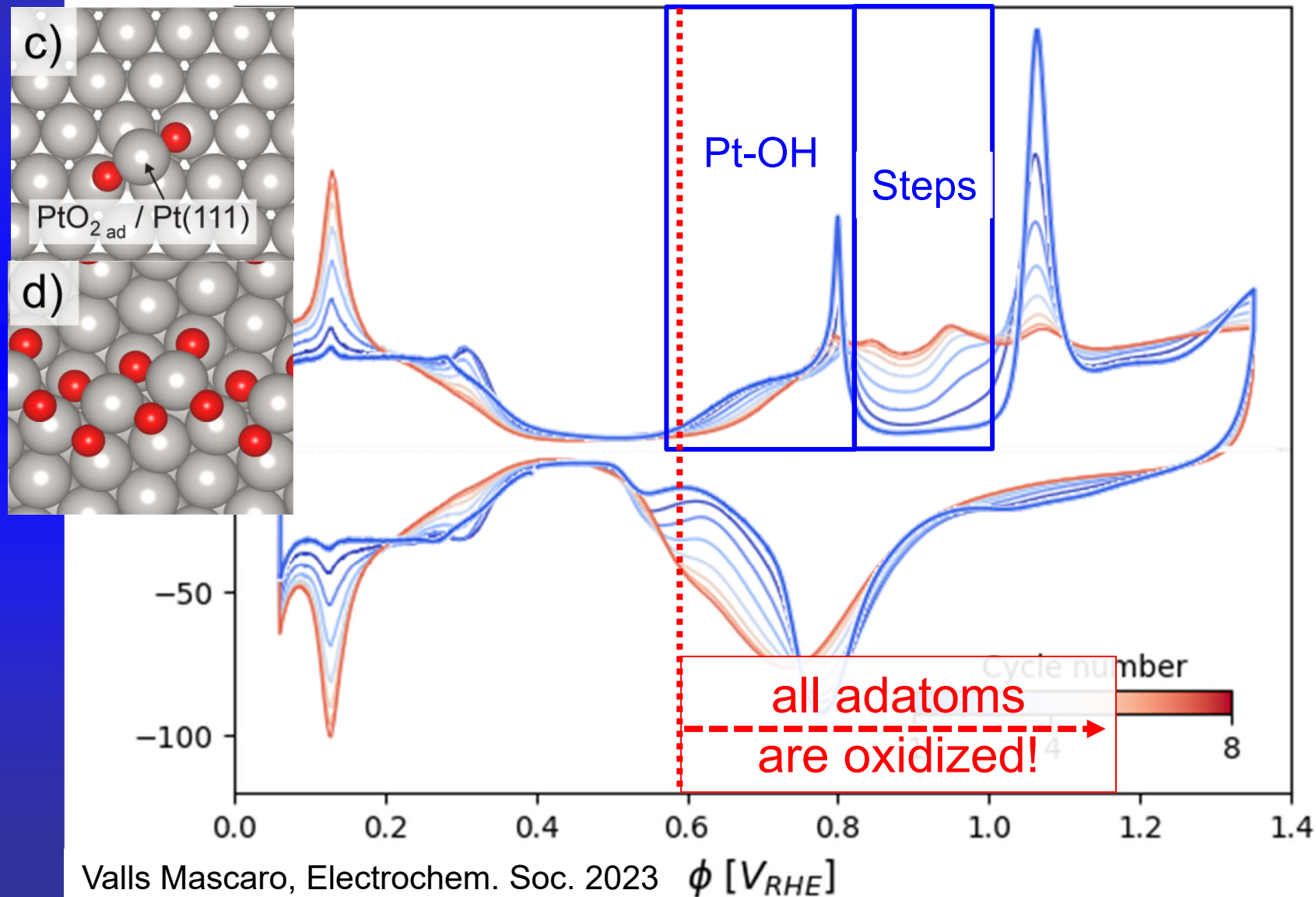
Pt(111) in 0.1 M $HClO_4$, with 50 mV/s and $[0.06; 1.35]$ V_{RHE}



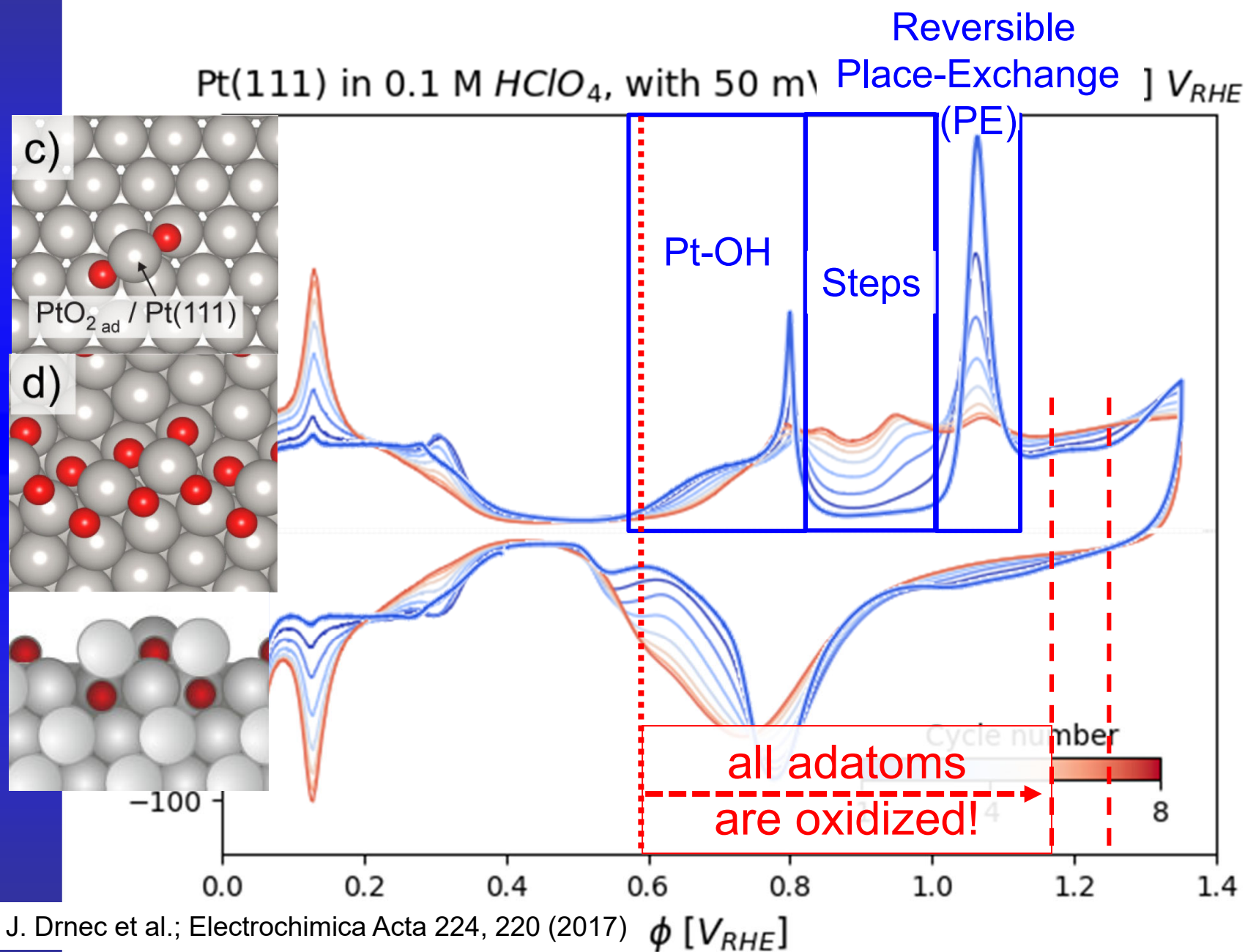
Valls Mascaro, Electrochem. Soc. 2023 ϕ [V_{RHE}]

All the Oxides...

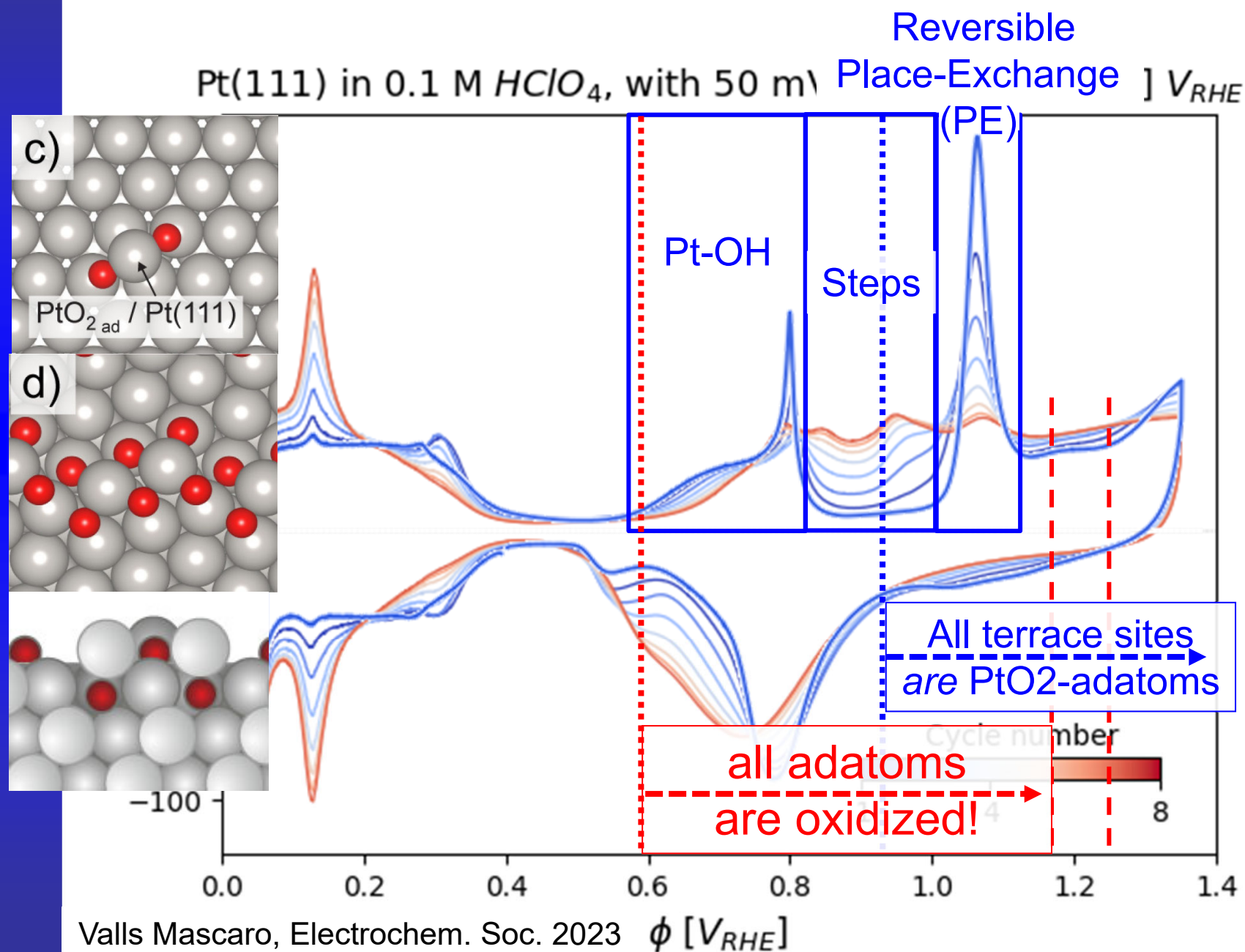
Pt(111) in 0.1 M $HClO_4$, with 50 mV/s and $[0.06; 1.35] V_{RHE}$



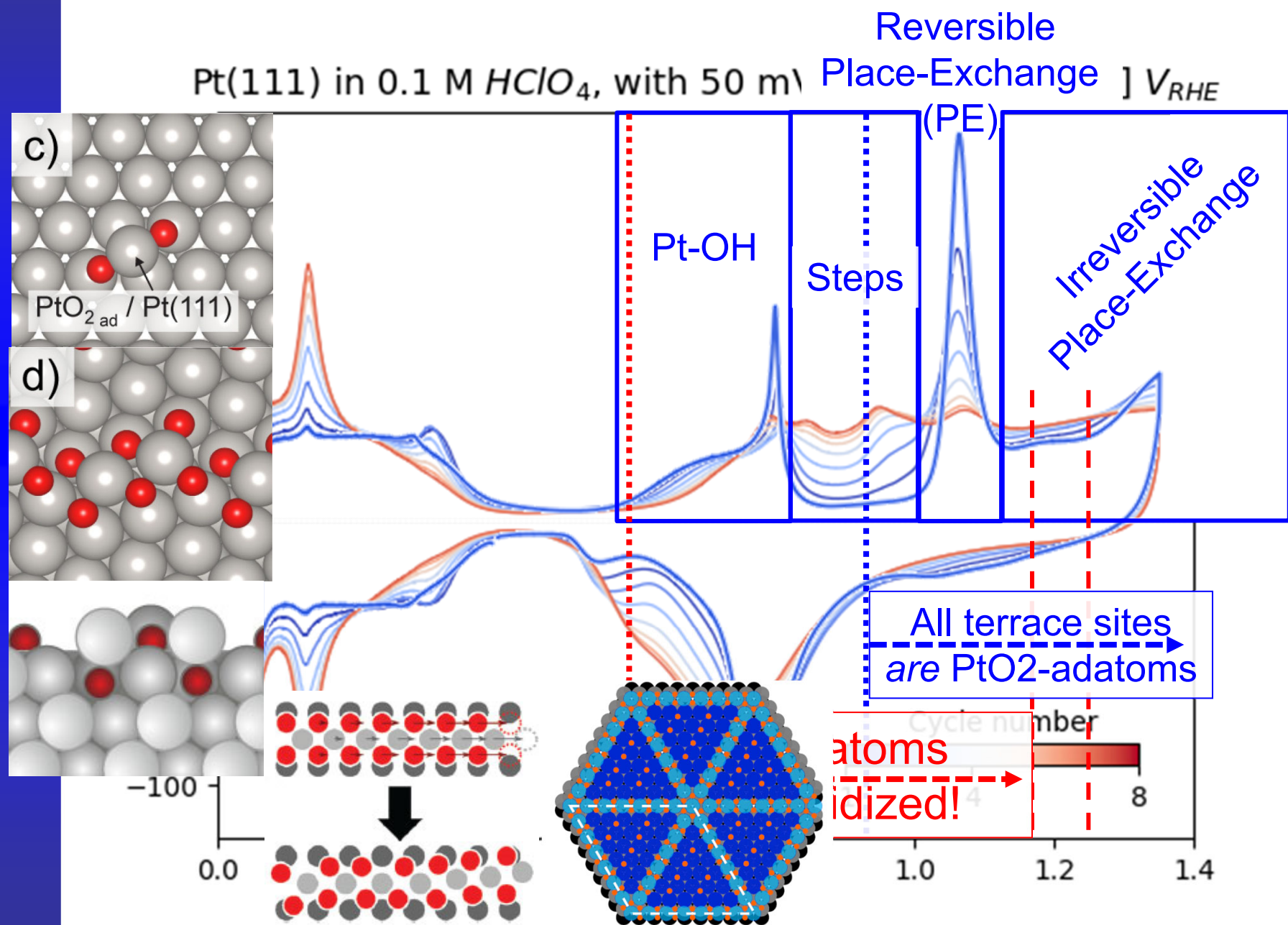
All the Oxides Structures...



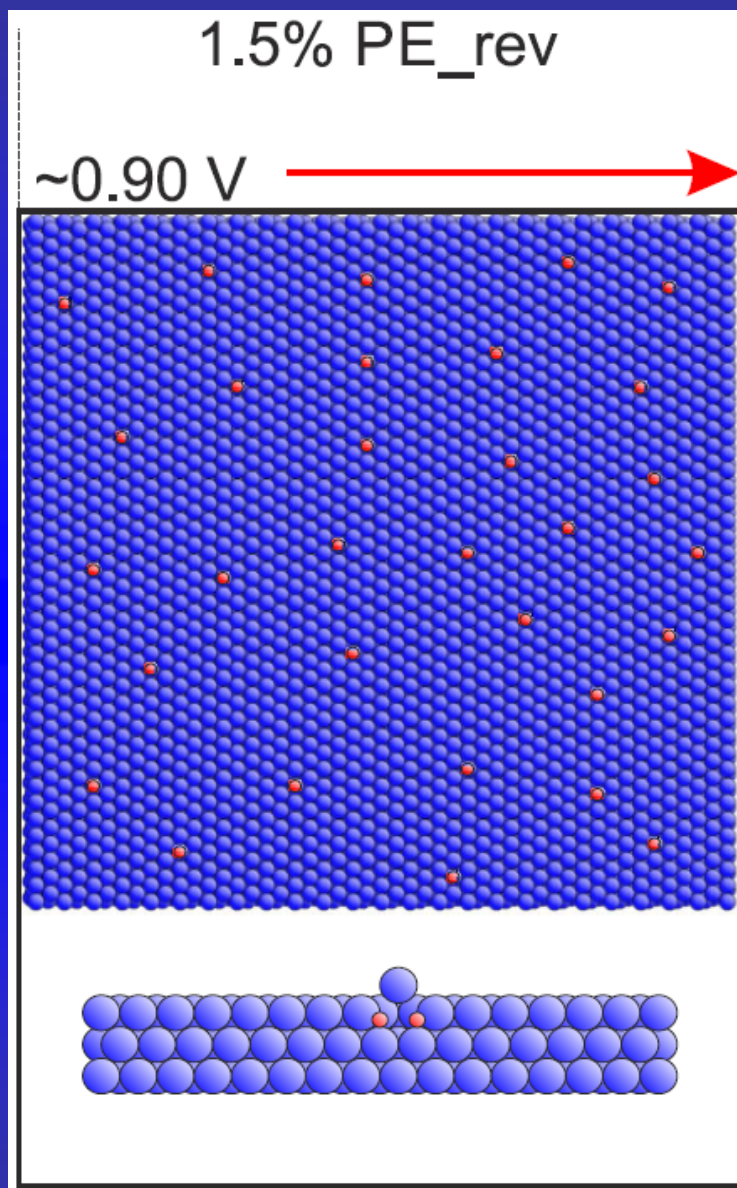
All the Oxides Structures...



All the Oxides Structures...



Pt(111) Oxidation: Reversible PE Adatom Gas



repulsive interaction

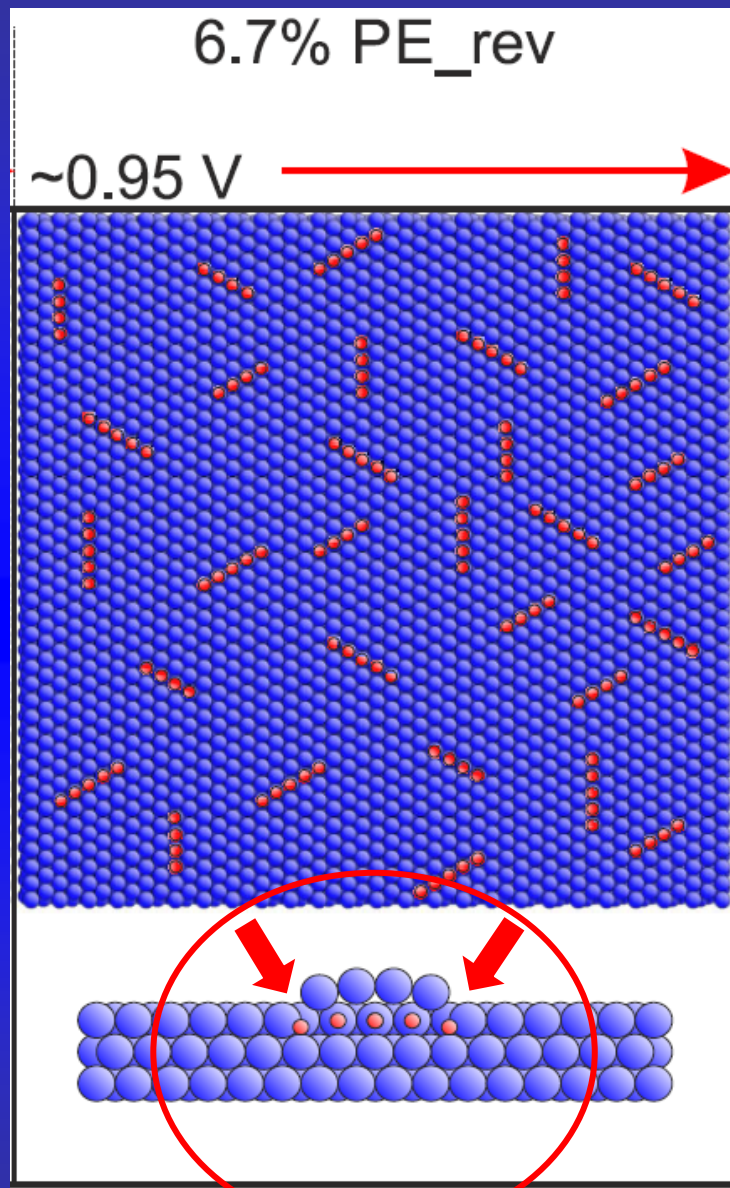
=>

reversible *Place Exchange Atoms*

maximize distances

M.J. Rost, L. Jacobse, M.T.M. Koper, *Angewandte Chemie* 2023

Pt(111) Oxidation: Formation of 1D Chains / Spokes



Reversible PE atom density high

=>

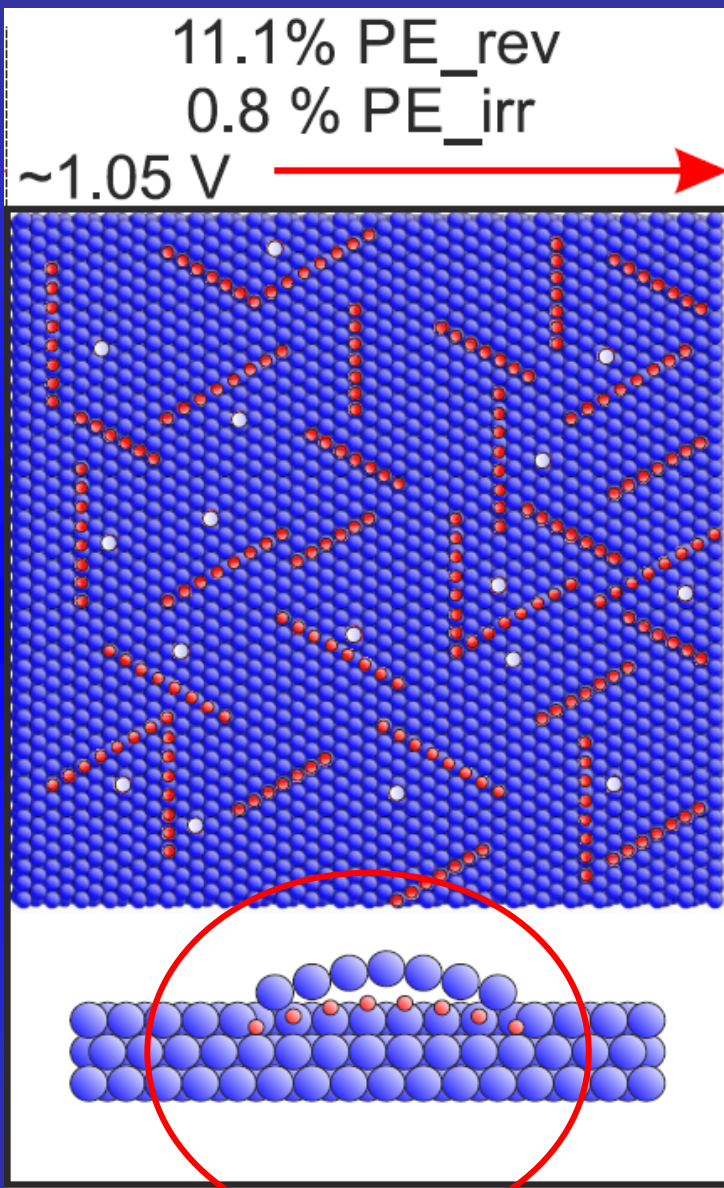
energetically more favorable to form
1D rows (or spokes).

rows resemble a α -PtO₂ structure
with larger lattice constant than Pt
=> stress is generated

stress partially reduced by a buckling,
but at ends rows push into surface

M.J. Rost, L. Jacobse, M.T.M. Koper, Angewandte Chemie 2023

Pt(111) Oxidation: Growth of Rows / Spokes



7 α -PtO₂ units exactly match 8 platinum units
=> stress relaxed by pushing out one atom

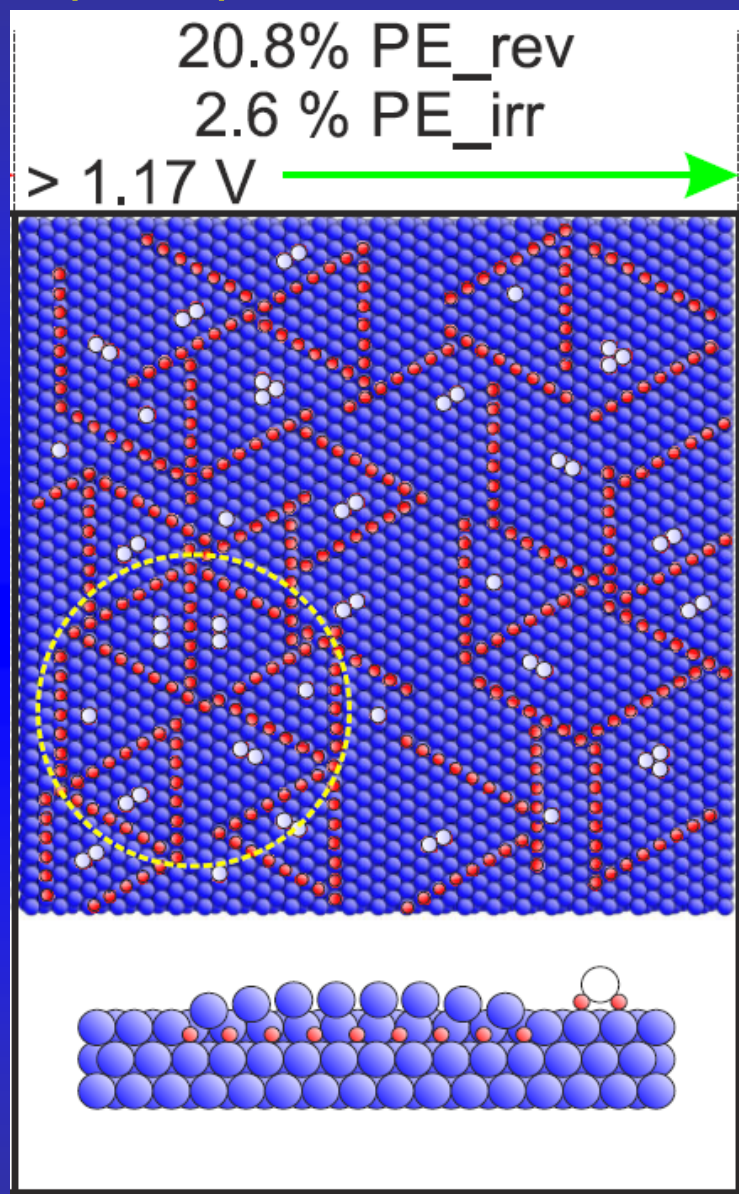
cross section still shows stressed row

entropy

=> rows align along all 3 orientations

M.J. Rost, L. Jacobse, M.T.M. Koper, Angewandte Chemie 2023

Pt(111) Oxidation: Creation of Irreversible PE Atoms

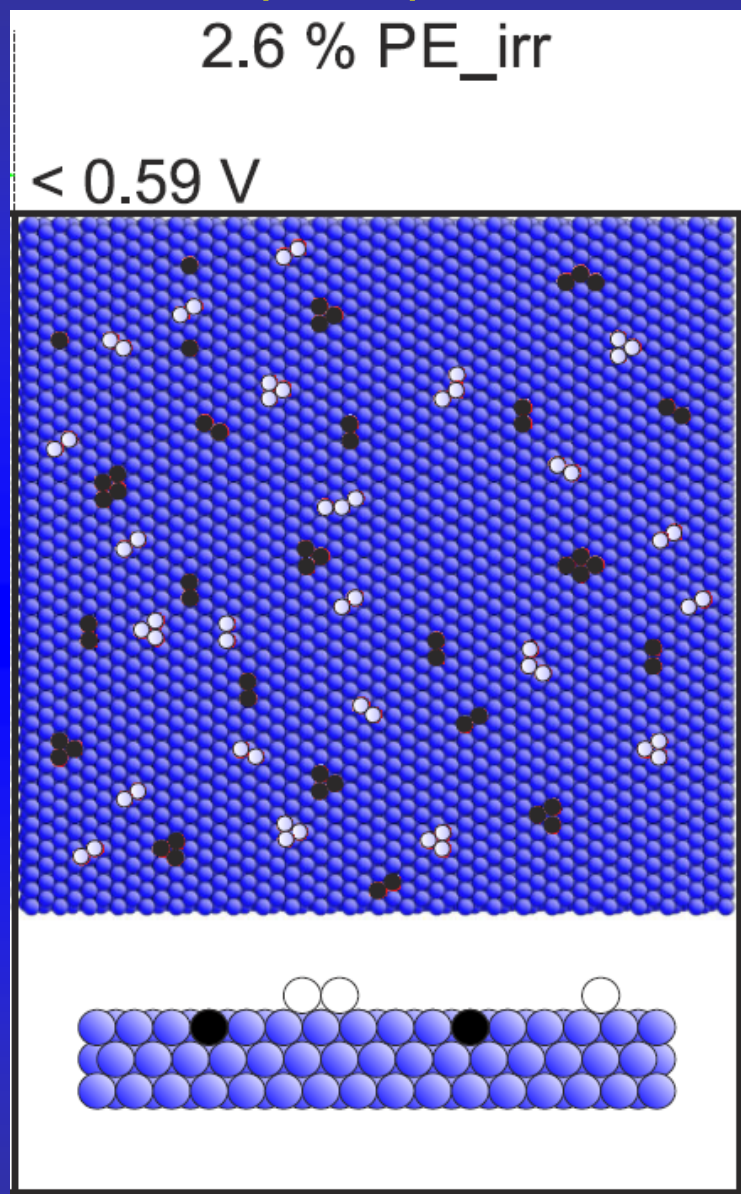


all rows reached critical lengths,
all rows pushed out one atom

disordered network of rows appears
type of spoke wheel structure
=> serves as the nucleation sites
that *a priori* did not exist

cross section shows a relaxed row
one platinum atom is pushed onto surface

Pt(111) Reduction: Adatoms & Vacancies



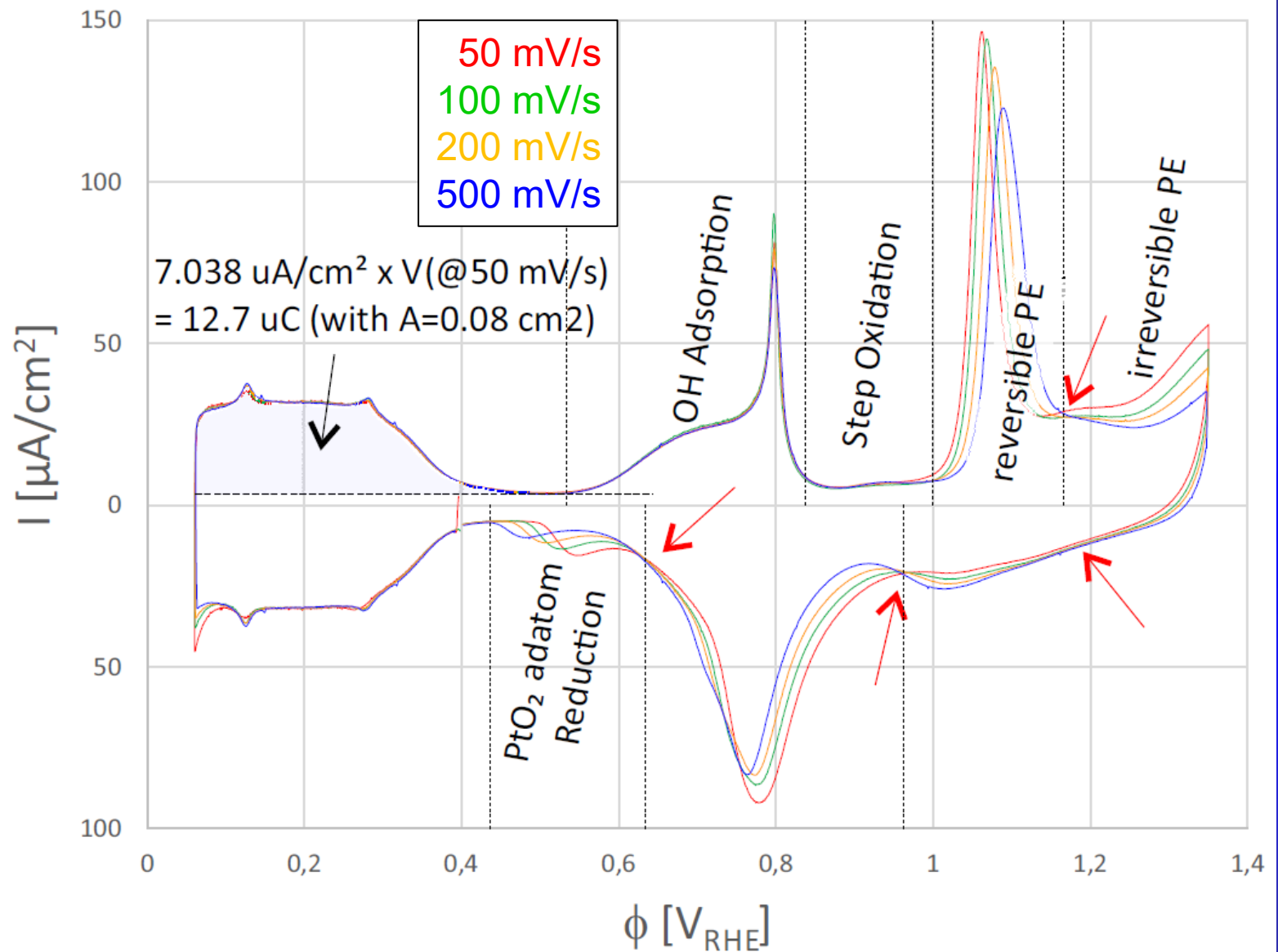
reduction: oxygen atoms are removed
reversible Place Exchange Atoms fall back
into their holes

pushed out atoms remain
as well as their holes

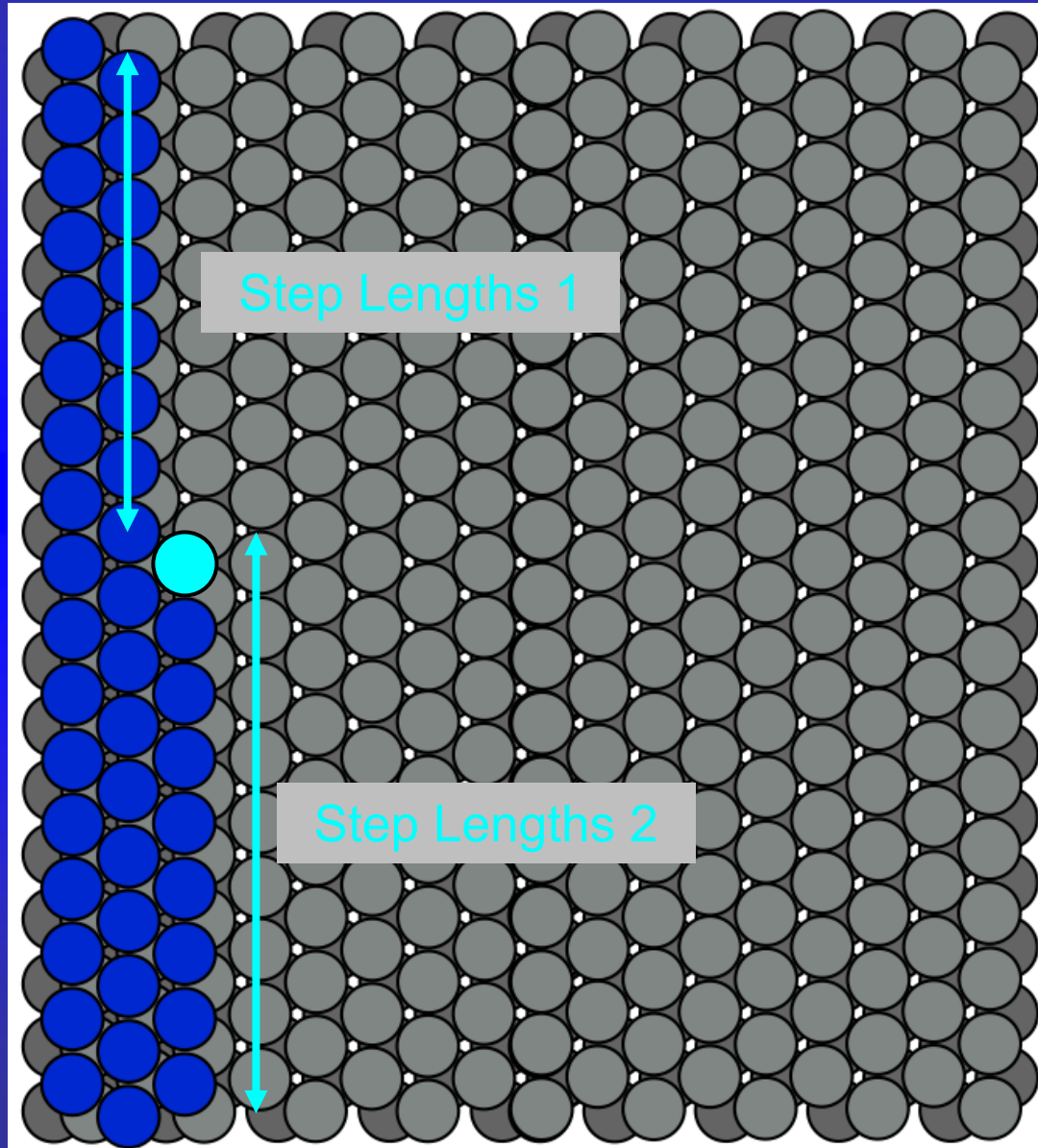
Nucleation and growth occurs
=> surface changes,

which is (one of) the reasons for the
deterioration of the electrode in a fuel cell.

What to Learn from Scan Rate Dependent Measurements ?

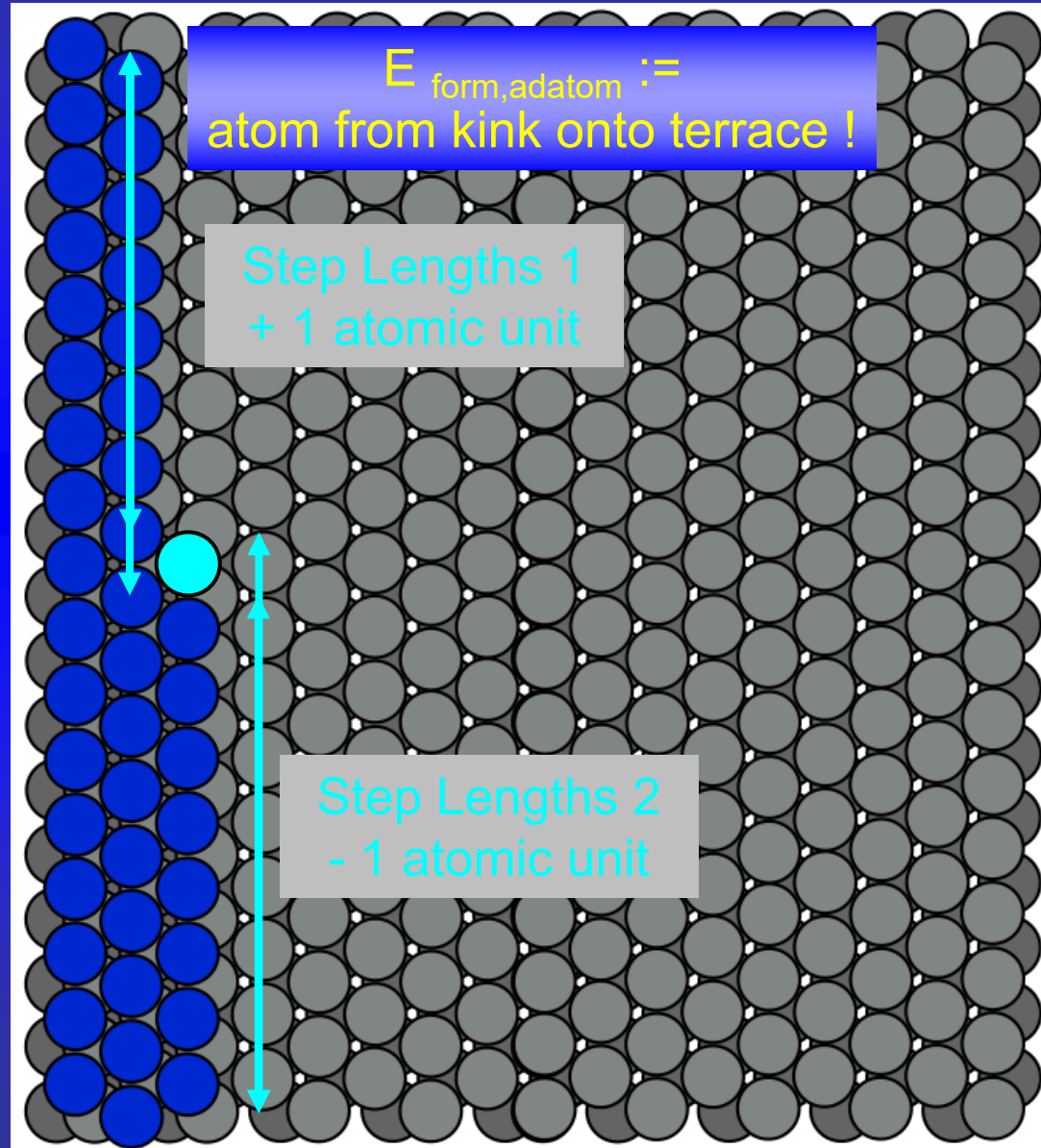


Kink = Very Special Site!



Kink Defines Adatom Formation Energy!

Step Lengths = Step Energy = const.



Chemical Potentials & Adatom Pressure:

$$\mu_{ad} = \underbrace{\mu_0}_{E_{\text{form,ad.}}} + \underbrace{k_B T \ln \left(\frac{\theta_{ad}}{1 - \theta_{ad}} \right)}_{\text{Entropy}} + \underbrace{W(\theta_{ad})}_{\text{Interaction}} \approx \mu_{ad} = \mu_0 + k_B T \ln(\theta_{ad})$$

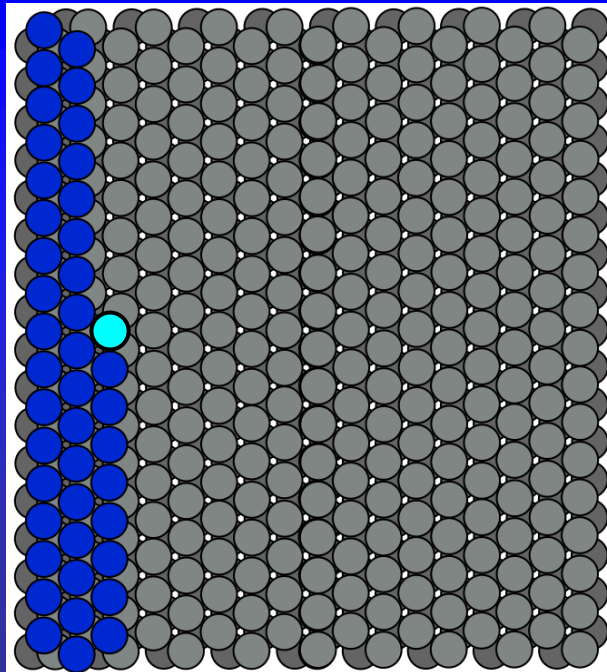
$E_{\text{form,ad.}}$

Entropy

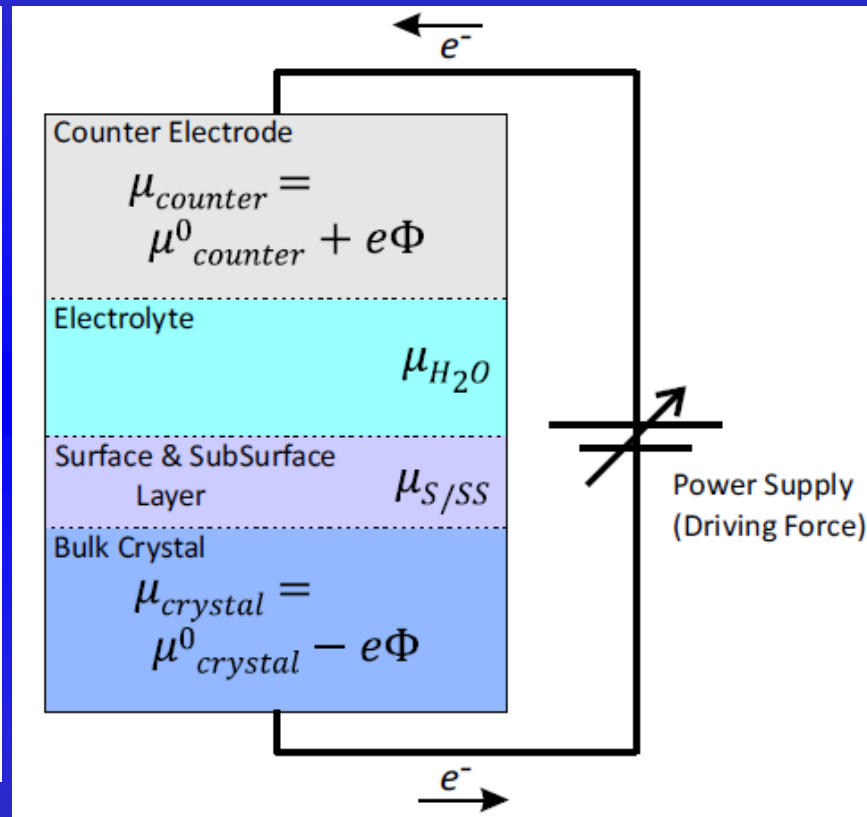
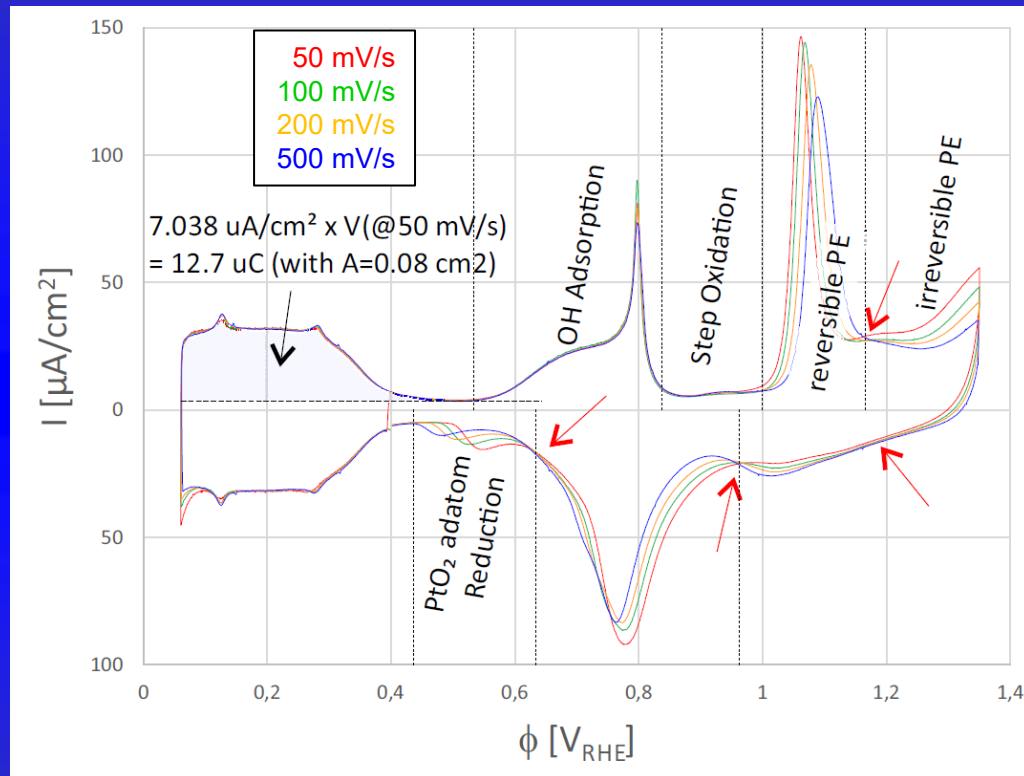
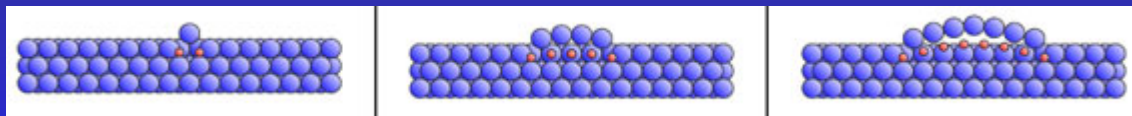
Interaction

Adatom Pressure follows
Boltzmann Distribution

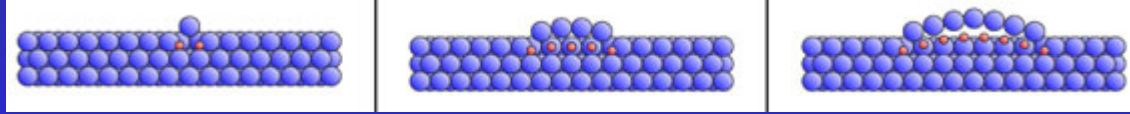
$$\theta_{ad} \approx \exp(-E_{\text{form,ad.}}/k_B T)$$



Arrhenius follows Frumkin to describe Atomic Diffusion involved Peaks CVs:



Arrhenius follows Frumkin to describe Atomic Diffusion involved Peaks CVs:

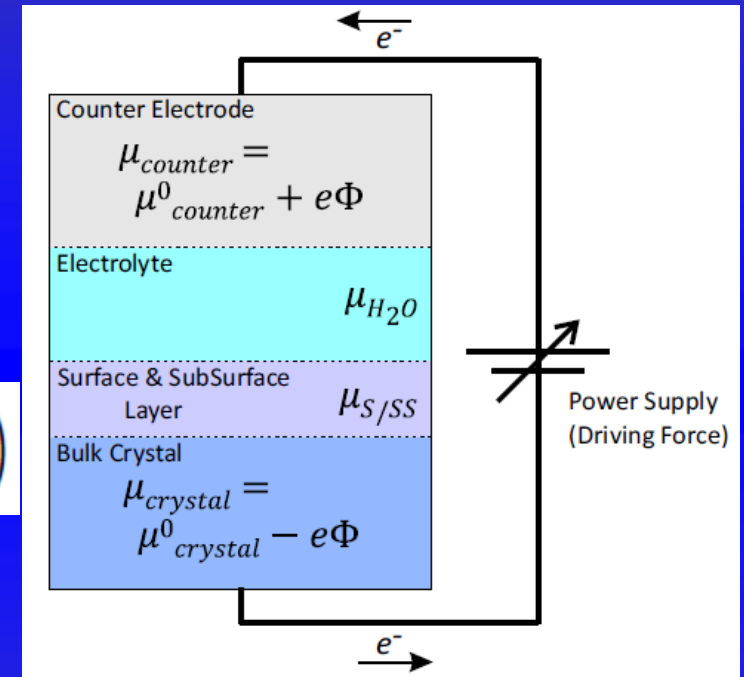


$$\mu_{H_2O} = \mu_{S/SS} = \mu_{crystal}$$

$$\mu_{H_2O} = \mu_{H_2O}^0(T) + k_B T \ln \left(\frac{c_{H_2O}}{c_O} \right)$$

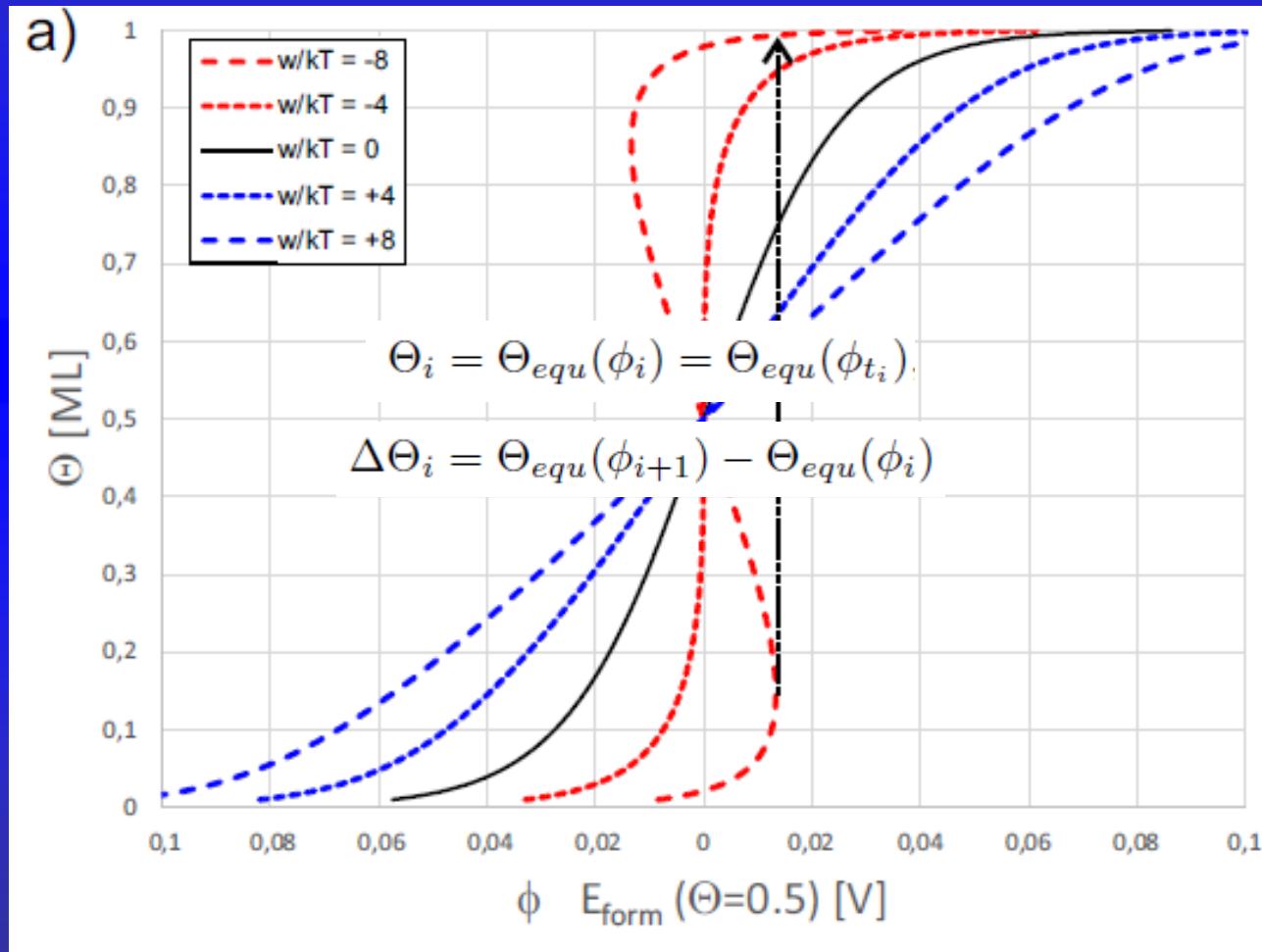
$$\mu_{S/SS} = \mu_{S/SS}^0(T) - ze(|\phi - \phi_{PZC}|) + w\Theta + k_B T \ln \left(\frac{\Theta}{1 - \Theta} \right)$$

$$\mu_{crystal} = \mu_{crystal}^0(T) - ze(\phi - \phi_{PZC})$$



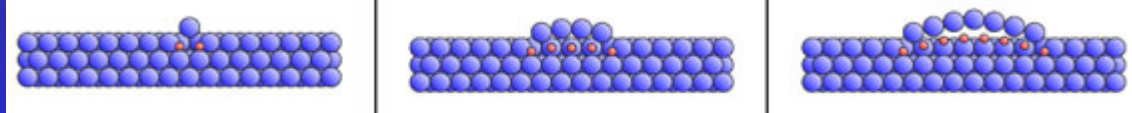
Arrhenius follows Frumkin to describe Atomic Diffusion involved Peaks CVs:

$$\mu_{S/SS} = \mu_{S/SS}^0(T) - ze(|\phi - \phi_{PZC}|) + w\Theta + k_B T \ln \left(\frac{\Theta}{1 - \Theta} \right)$$



Delay due to Atomic Diffusion:

Pt atom has to diffuse up, while 2 oxygen's diffuse subsurface



$$\Theta_i = \Theta_{equ}(\phi_i) = \Theta_{equ}(\phi_{t_i})$$

$$\Delta\Theta_i = \Theta_{equ}(\phi_{i+1}) - \Theta_{equ}(\phi_i)$$

$$\Delta\Theta_i = (\Theta_{equ}(\phi_{i+1}) - \Theta_{act}(\phi_i)) \times \Delta t * \nu_0 * \exp(-E_{diff}/kT)$$

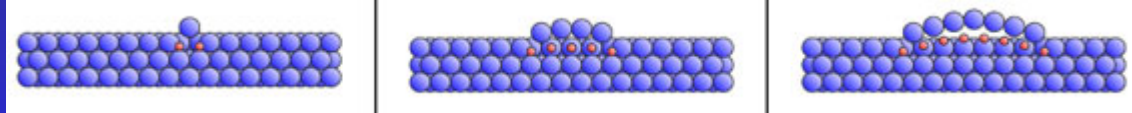
$$\Theta_{act} = \sum_i \Delta\Theta_i$$

4 fit parameters:

E_{form}^0 , w , ν_0 , E_{diff}^0

Delay due to Atomic Diffusion:

Pt atom has to diffuse up, while 2 oxygen's diffuse subsurface



$$\Theta_i = \Theta_{equ}(\phi_i) = \Theta_{equ}(\phi_{t_i})$$

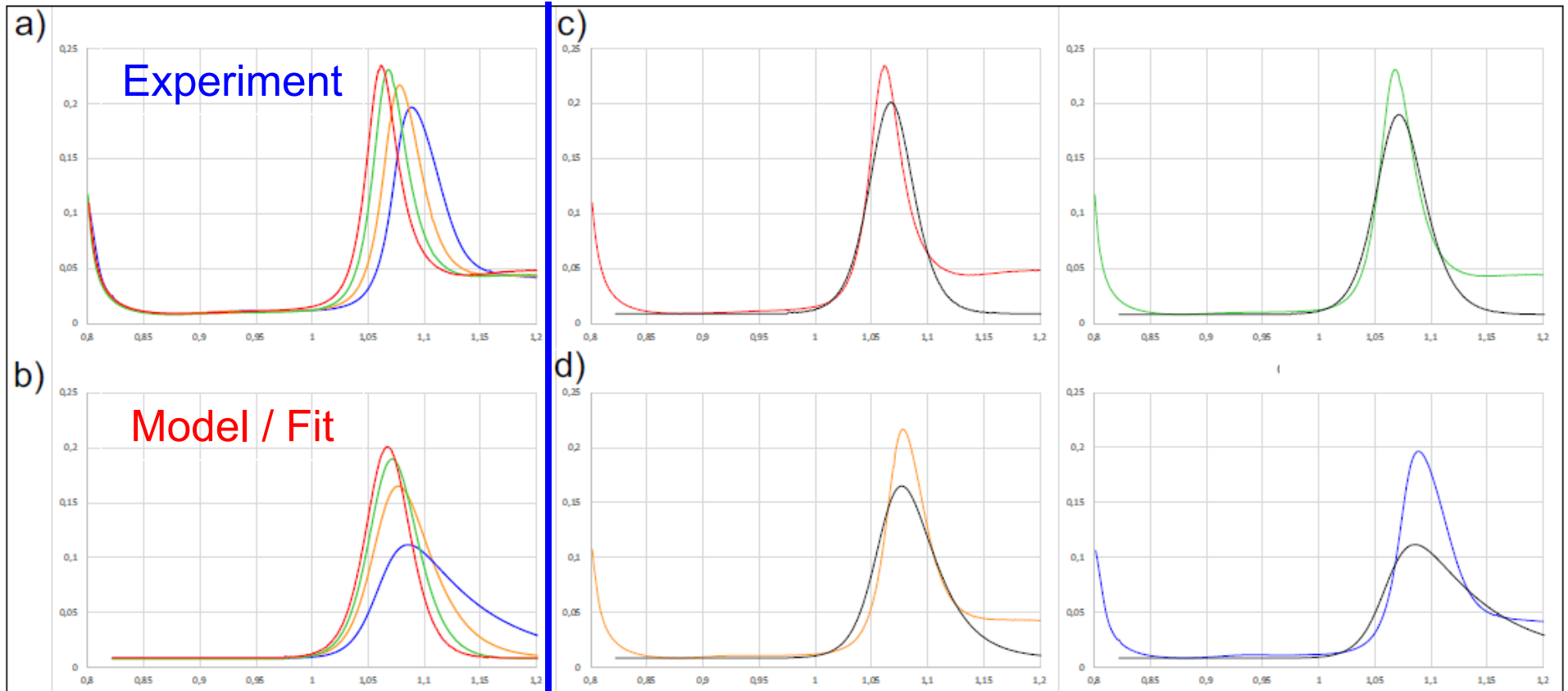
$$\Delta\Theta_i = \Theta_{equ}(\phi_{i+1}) - \Theta_{equ}(\phi_i)$$

$$\frac{d\Theta}{dt} = \left(\Theta_{equ}(\phi(t)) - \int_0^t \frac{d\Theta}{dt} \right) \times \Delta t \nu_0 \exp(- (E_{diff}) / kT)$$

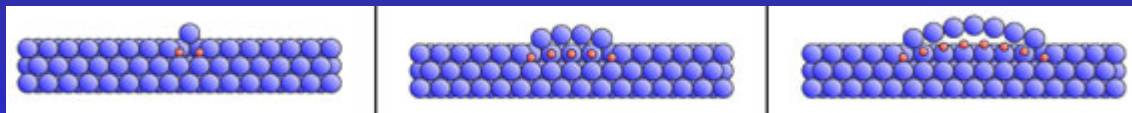
4 fit parameters:

E_{form}^0 , w , ν_0 , E_{diff}^0

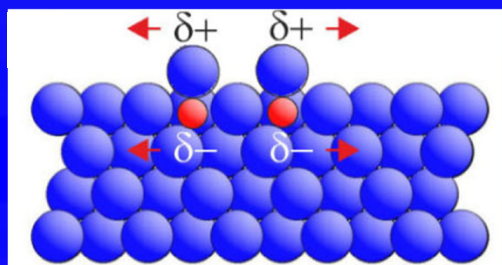
4 parameter numerical fit: E^0_{form} , w , v_0 , E^0_{diff}



Accounting for Dipole Moments thus Potential Dependence: Brønsted–Evans–Polanyi (BEP)



$$\frac{d\Theta}{dt} = \left(\Theta_{equ}(\phi(t)) - \int_0^t \frac{d\Theta}{dt} \right) \times \Delta t \nu_0 \exp(- (E_{diff}) / kT)$$



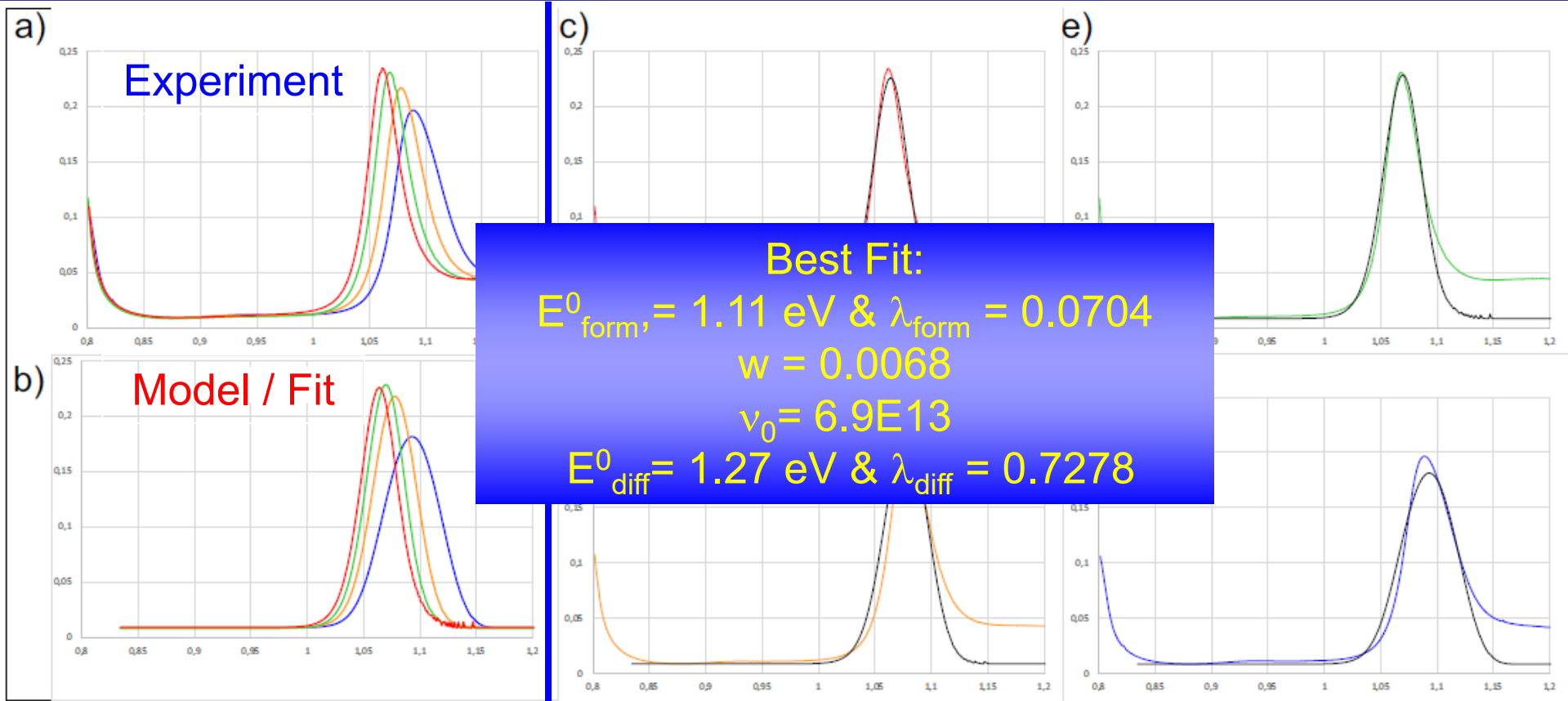
$$E_{form} = E_{form}^0 - e\lambda_{form}(\phi - \phi_{PZC})$$

$$E_{diff} = E_{diff}^0 - e\lambda_{diff}(\phi - \phi_{PZC}),$$

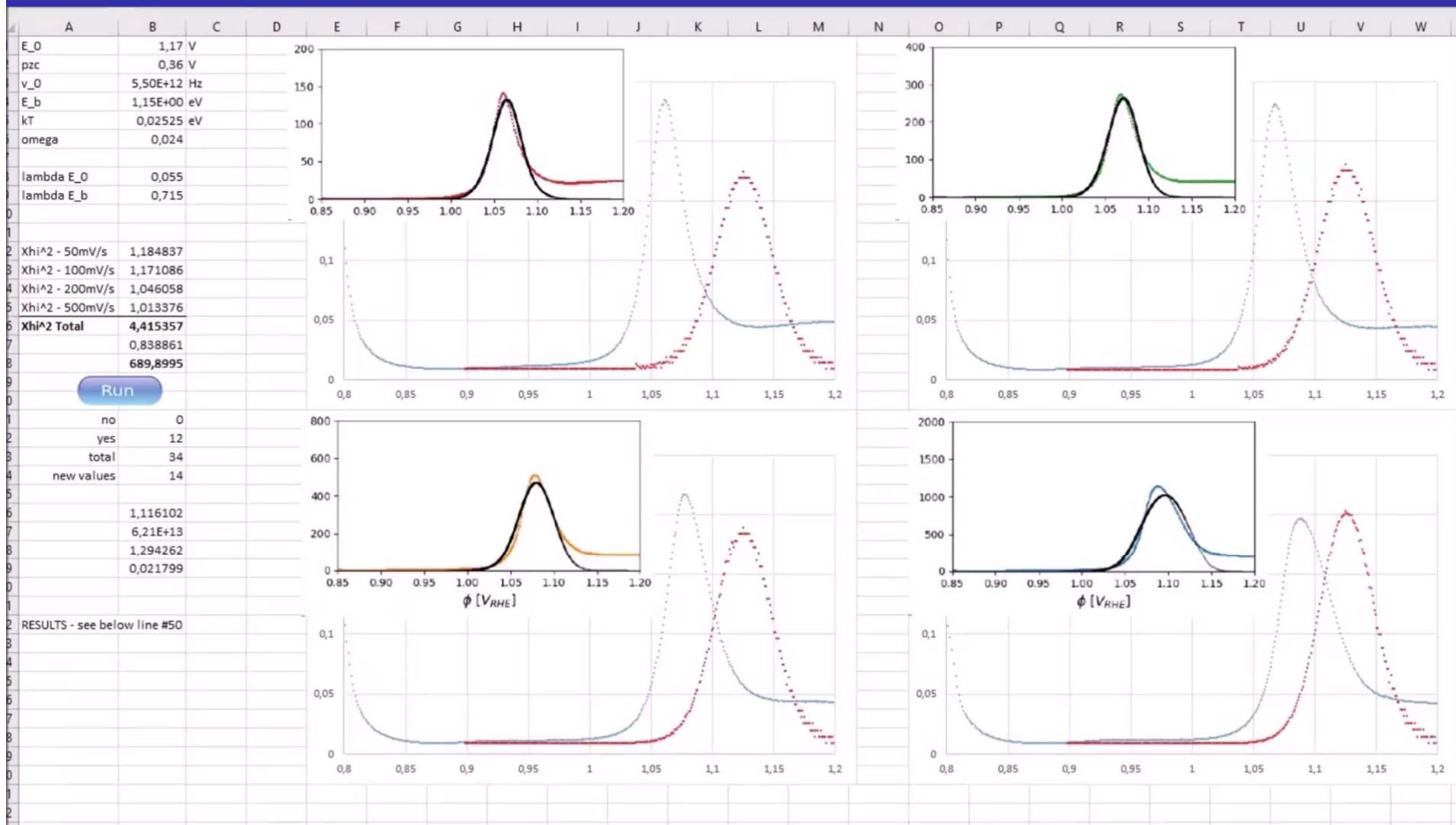
6 fit parameters:

$$E_{form}^0, \lambda_{form}, w, \nu_0, E_{diff}^0, \lambda_{diff}$$

6 parameter num. fit: $E^0_{\text{form}}, \lambda_{\text{form}}, w, \nu_0, E^0_{\text{diff}}, \lambda_{\text{diff}}$



Kinetic Monte Carlo Approach on χ^2 :



6 parameter fit: E^0_{form} , λ_{form} , w , ν_0 , E^0_{diff} , λ_{diff}

Best Fit:

$$E^0_{\text{form}} = 1.11 \text{ eV} \ \& \ \lambda_{\text{form}} = 0.0704$$

$$w = 0.0068$$

$$\nu_0 = 6.9\text{E}13$$

$$E^0_{\text{diff}} = 1.27 \text{ eV} \ \& \ \lambda_{\text{diff}} = 0.7278$$

GDCh

Research Articles

Angewandte
International Edition
Chemie
www.angewandte.org

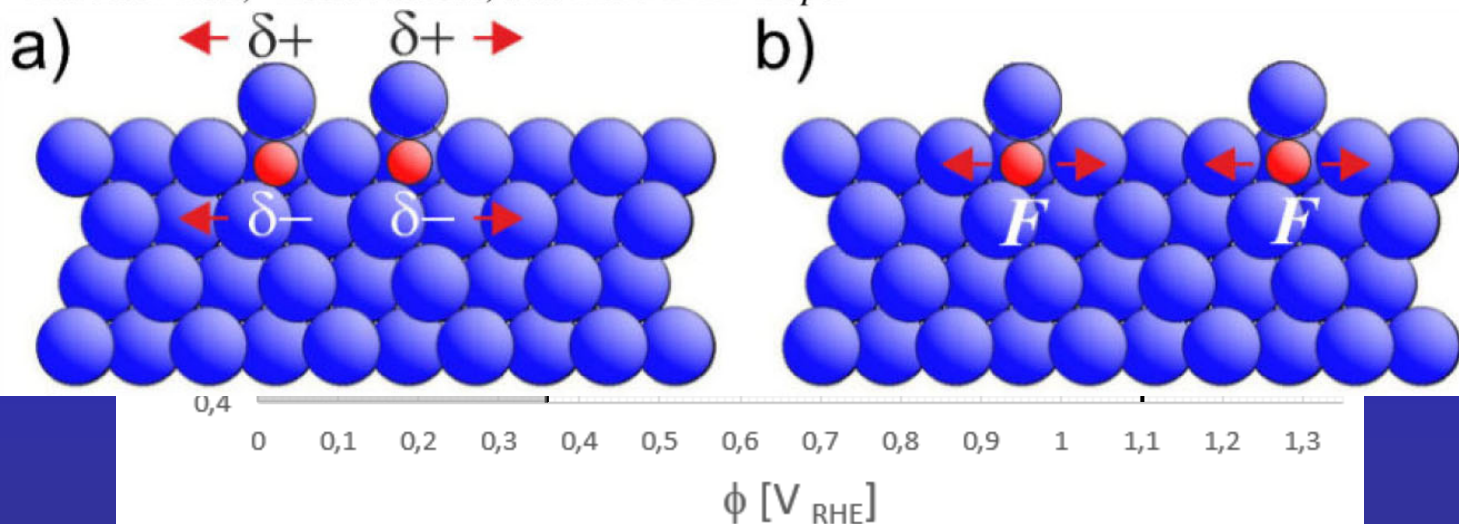
How to cite:

doi.org/10.1002/anie.202216376

Electrochemistry

Non-Random Island Nucleation in the Electrochemical Roughening on Pt(111)

Marcel J. Rost,* Leon Jacobse, and Marc T. M. Koper



94)

6 parameter fit: E^0_{form} , λ_{form} , w , ν_0 , E^0_{diff} , λ_{diff}

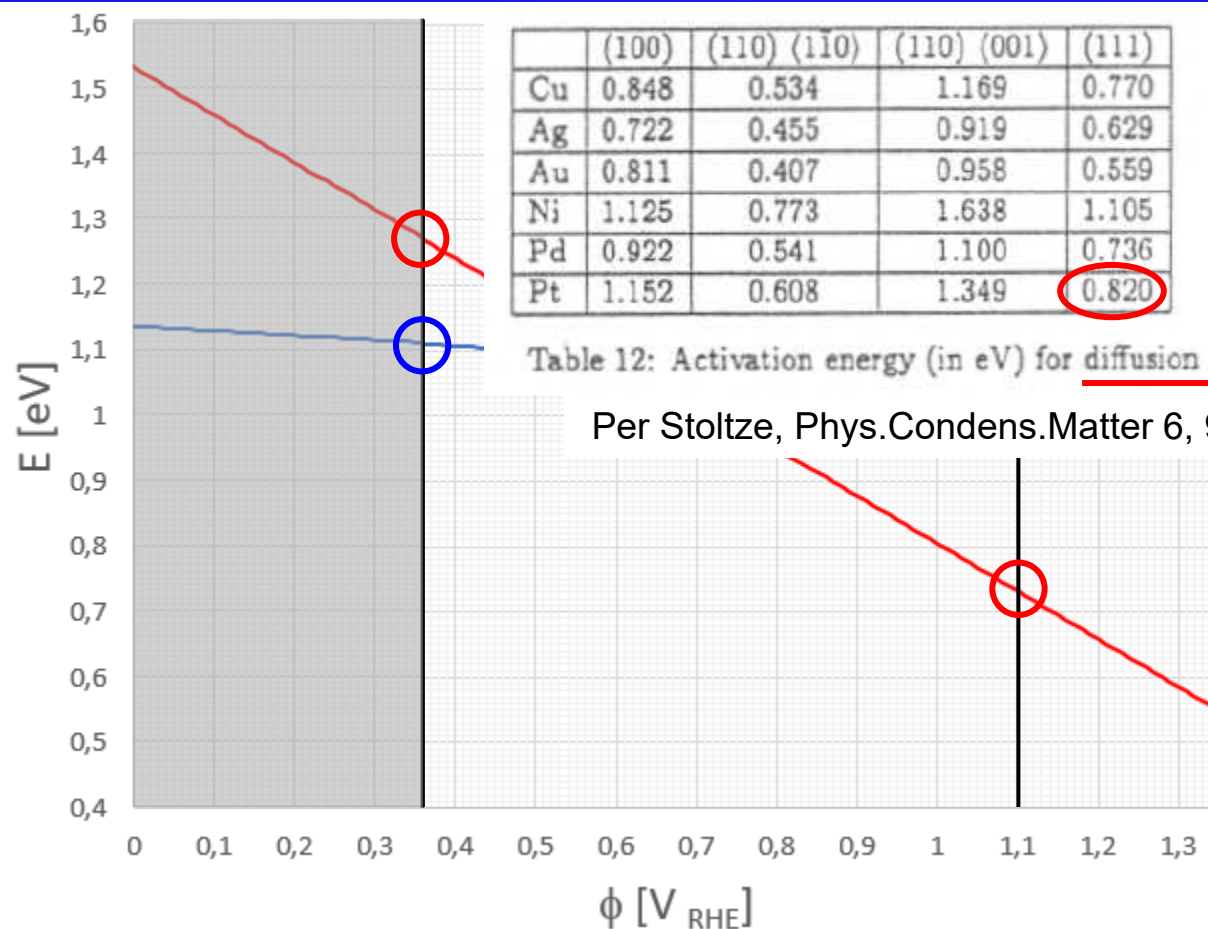
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$$E^0_{\text{diff}} = 1.27 \text{ eV} \ \& \ \lambda_{\text{diff}} = 0.7278$$



6 parameter fit: E^0_{form} , λ_{form} , w , ν_0 , E^0_{diff} , λ_{diff}

Best Fit:

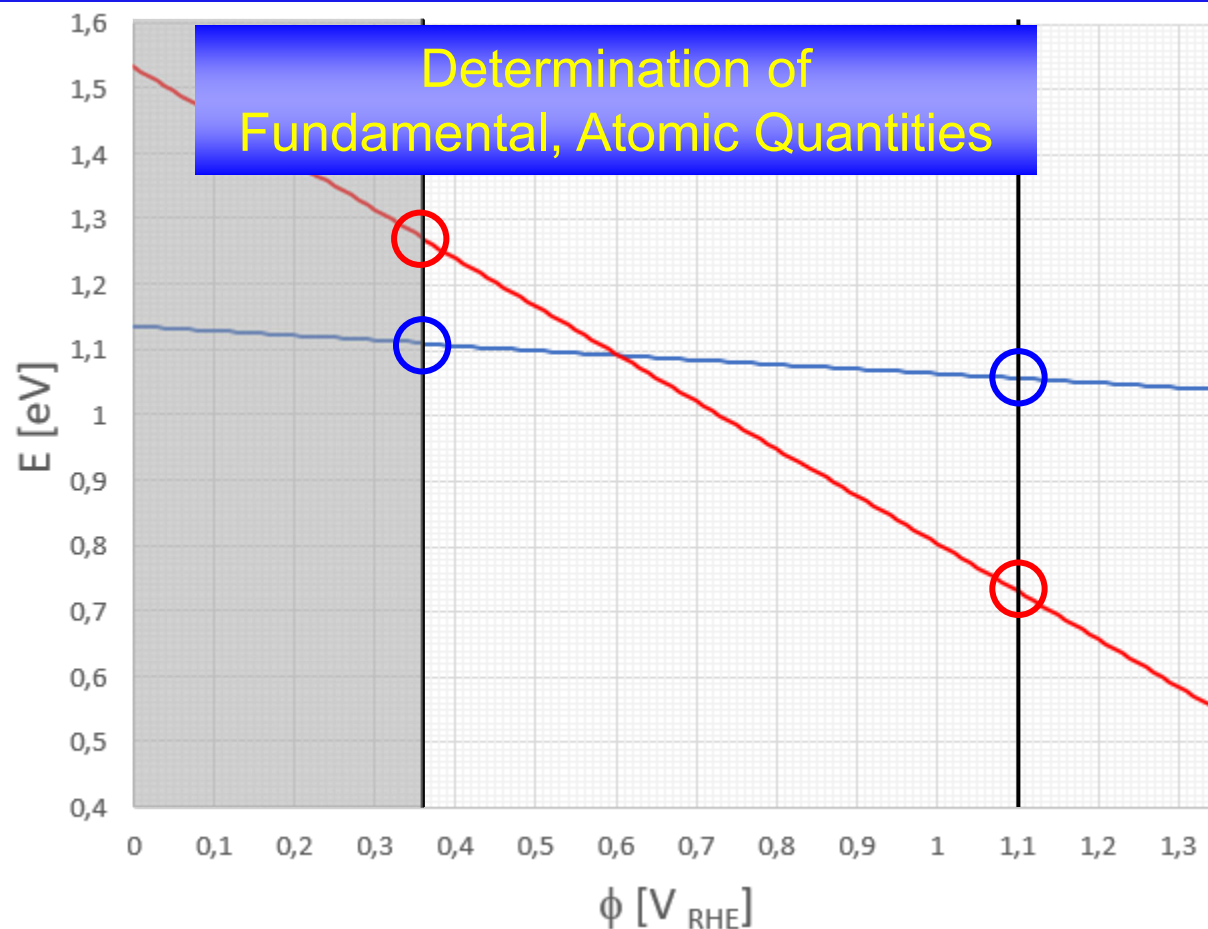
$$E^0_{\text{form}} = 1.11 \text{ eV} \ \& \ \lambda_{\text{form}} = 0.0704$$

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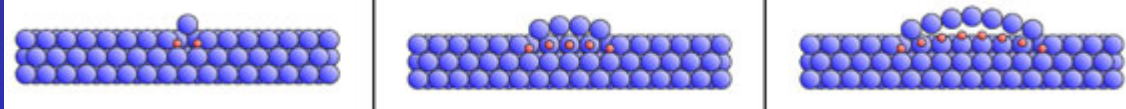
$$\nu_0 = 6.9\text{E}13$$

$$E^0_{\text{diff}} = 1.27 \text{ eV} \ \& \ \lambda_{\text{diff}} = 0.7278$$

Determination of
Fundamental, Atomic Quantities



towards an analytical fit:



$$\frac{d\Theta}{dt} = \left(\Theta_{equ}(\phi(t)) - \int_0^t \frac{d\Theta}{dt} \right) \times \Delta t \nu_0 \exp(- (E_{diff}) / kT)$$

$$E_{form} = E_{form}^0 - e\lambda_{form} (\phi - \phi_{PZC})$$

$$E_{diff} = E_{diff}^0 - e\lambda_{diff} (\phi - \phi_{PZC}),$$

6 fit parameters:

$$E_{form}^0, \lambda_{form}, w, \nu_0, E_{diff}^0, \lambda_{diff}$$

$$0 = \mu_{S/SS}^0(T) - \mu_{S/SS} - ze(|\phi - \phi_{PZC}|) + w\Theta + k_B T \ln \left(\frac{\Theta}{1 - \Theta} \right)$$

$$0 = E_{form} + w\Theta + k_B T \ln \left(\frac{\Theta}{1 - \Theta} \right)$$

$$0 = E_{form}^0 - e\lambda_{form} (\phi - \phi_{PZC}) + w\Theta + k_B T \ln \left(\frac{\Theta}{1 - \Theta} \right)$$

$$\Theta_{equ} = \gamma / (1 + \gamma) - \frac{w}{k_B T} \times \gamma^2 / (1 + \gamma)^3$$

$$\gamma := \exp(-E_{form}(\phi) / kT)$$

after quite some math...:

$$\lambda e \phi = \underbrace{\mu_{ys}^{\circ}(T) - \mu_{K_2O}^{\circ}(T)}_{\equiv E_{form}} + w\theta + k_B T \ln\left(\frac{\theta}{1-\theta}\right)$$

$$\Rightarrow -\frac{1}{k_B T} E_{form}(\phi) = \ln\left(e^{w\theta/k_B T}\right) + \ln\left(\frac{\theta}{1-\theta}\right)$$

, where $E_{form}(\phi) = E_{form} - \lambda e \phi$

$$\Rightarrow \exp\left(-E_{form}(\phi)/k_B T\right) = \left(e^{w\theta/k_B T}\right)\left(\frac{\theta}{1-\theta}\right)$$

, here we assume $w/k_B T < 0.1$ and $0 \leq \theta \leq 1$.

Then we can make a Taylor expansion on the right hand side:

$$e^{w\theta/k_B T} \approx 1 + \frac{w\theta}{k_B T} + \mathcal{O}(\theta^2)$$

$\Rightarrow$ and denote: $\gamma = \exp(-E_{form}(\phi)/k_B T)$

$$\Rightarrow (1-\theta)\gamma = \left(1 + \frac{w\theta}{k_B T}\right)\theta$$

$$\Rightarrow \left(\frac{w}{k_B T}\right)\theta^2 + (1+\gamma)\theta - \gamma = 0$$

$\Rightarrow$ quadratic in θ , thus solvable, which gives:

$$\theta = \frac{-(1+\gamma) \pm \sqrt{(1+\gamma)^2 + 4\left(\frac{w}{k_B T}\right)\gamma}}{2w/k_B T}$$

, only the solution with the + sign is physical.

Moreover, as $w/k_B T < 1$, we can make a Taylor expansion of the root: $\sqrt{1+x} \approx 1 + \frac{x}{2} - \frac{x^2}{8} + O(x^3)$

$$\Rightarrow \left(\frac{2w}{k_B T}\right) \cdot \theta \approx -(1+\gamma) + (1+\gamma) \left[1 + \left(\frac{2w}{k_B T}\right) \gamma / (1+\gamma)^2 - 2 \left(\frac{w}{k_B T}\right)^2 \frac{\gamma^2}{(1+\gamma)^3} \right]$$
$$= \left(\frac{2w}{k_B T}\right) \gamma / (1+\gamma) - 2 \left(\frac{w}{k_B T}\right)^2 \frac{\gamma^2}{(1+\gamma)^3}$$

$\Rightarrow$

$$\theta_{eq} = \gamma / (1+\gamma) - (w/k_B T) \gamma^2 / (1+\gamma)^3 \quad \square$$

This is the equilibrium coverage.

The coverage will be driven by:

$$\dot{\theta} = [\theta_{eq}(\phi) - \theta] v_0 e^{-[E_d - \alpha \phi] \beta}$$

, where $\beta = 1/k_B T$

, now $\phi = \phi_0 + s \cdot t$ with sweep rate s .

$$d\phi = s \cdot dt$$

$$\Rightarrow \frac{\partial \theta}{\partial \phi} \cdot \frac{\partial \phi}{\partial t} = [\theta_{eq}(\phi) - \theta] v_0 e^{-[E_d - \alpha \phi] \beta}$$

$\Rightarrow$

$$\frac{\partial \theta}{\partial \phi} \cdot s = [\theta_q(\phi) - \theta] v_0 e^{-(\tilde{E}_0 - \alpha \phi) \beta}$$

$$= [\theta_q(\phi) - \theta] \tilde{v}_0 e^{\alpha \beta \phi}$$

$$\tilde{v}_0 = v_0 e^{-\beta E_0^0}$$

$$\theta_q = \gamma / (1 + \gamma) - (w / a_0 T) \gamma^2 / (1 + \gamma)^3$$

$$\text{with } \gamma = \exp(-E_{\text{form}}(\phi) / a_0 T)$$

$$= \exp(-\beta E_{\text{form}}^0 + \alpha' \beta \phi)$$

$$u = e^{\eta \phi} \Rightarrow du = \eta e^{\eta \phi} d\phi = \eta u d\phi$$

$$\phi = \frac{1}{\eta} \ln(u)$$

$$\Rightarrow \frac{\partial \theta}{\partial \phi} = \frac{\partial \theta}{\partial u} \cdot \frac{\partial u}{\partial \phi} = \eta u \frac{\partial \theta}{\partial u}$$

$$\frac{e^{\alpha \beta \phi}}{e^{\alpha' \beta \phi}} = \frac{e^{\frac{\alpha \beta}{\eta} \ln(u)}}{e^{\frac{\alpha' \beta}{\eta} \ln(u)}} = \frac{u^{\alpha \beta / \eta}}{u^{\alpha' \beta / \eta}}$$

$$\gamma = \tilde{\gamma}_0 \cdot u^{\alpha' \beta / \eta} \quad \text{with} \quad \tilde{\gamma}_0 = e^{-\beta E_{\text{form}}^0 \cdot \rho}$$

$$\Rightarrow \eta u \frac{\partial \theta}{\partial u} s = \left[\frac{\gamma}{(1 + \gamma)} - (w / a_0 T) \frac{\gamma^2}{(1 + \gamma)^3} - \theta \right] \cdot \tilde{v}_0 \cdot u^{\alpha \beta / \eta}$$

choice: $\eta = \alpha\beta$

Hence, we obtain the following differential equation:

$$\frac{\partial \theta}{\partial u} = \left[\frac{\gamma}{1+\gamma} - \left(\frac{u}{k_B T} \right) \frac{\gamma^2}{(1+\gamma)^3} - \theta \right] \cdot \frac{\tilde{v}_0}{\alpha \beta s}$$

let us have a closer look at the γ parameter:

$$\gamma = \tilde{\gamma}_0 u^{\alpha'/\alpha}, \text{ yet realize that } \alpha'/\alpha \ll 1.$$

which allows us to use the following approx.:

$$\gamma = \tilde{\gamma}_0 u^{\alpha'/\alpha} \approx \tilde{\gamma}_0 \left[1 + \underbrace{\left(\frac{\alpha'}{\alpha} \right)}_{\tilde{\alpha}} \log(u) \right]$$

, thus we find:

$$\frac{\gamma}{1+\gamma} \approx \frac{\tilde{\gamma}_0 [1 + \tilde{\alpha} \log(u)]}{[1 + \tilde{\gamma}_0] + \left\{ \frac{\tilde{\gamma}_0}{1 + \tilde{\gamma}_0} \right\} \tilde{\alpha} \log(u)}$$

$$\approx \left[\frac{\tilde{\gamma}_0}{1 + \tilde{\gamma}_0} \right] (1 + \tilde{\alpha} \log(u)) \left(1 - \left\{ \frac{\tilde{\gamma}_0}{1 + \tilde{\gamma}_0} \right\} \tilde{\alpha} \log(u) \right)$$

$$\approx \left[\frac{\tilde{\gamma}_0}{1 + \tilde{\gamma}_0} \right] \left\{ 1 + \tilde{\alpha} \log(u) \left[1 - \frac{\tilde{\gamma}_0}{1 + \tilde{\gamma}_0} \right] \right\}$$

→

$$\frac{\gamma}{1 + \gamma} \approx \frac{\tilde{\gamma}_0}{1 + \tilde{\gamma}_0} \left\{ 1 + \frac{\tilde{\alpha} \log(u)}{1 + \tilde{\gamma}_0} \right\}$$

$$\frac{\gamma^2}{(1 + \gamma)^3} \approx \frac{\tilde{\gamma}_0^2 [1 + \tilde{\alpha} \log(u)]^2}{(1 + \tilde{\gamma}_0 [1 + \tilde{\alpha} \log(u)])^3}$$

$$= \frac{\tilde{\gamma}_0^2 (1 + 2\tilde{\alpha} \log(u))}{([1 + \tilde{\gamma}_0] [1 + \frac{\tilde{\gamma}_0 \tilde{\alpha}}{1 + \tilde{\gamma}_0} \log(u)])^3}$$

$$= \frac{\tilde{\gamma}_0^2}{(1 + \tilde{\gamma}_0)^3} \cdot \frac{(1 + 2\tilde{\alpha} \log(u))}{\left\{ 1 + 3 \frac{\tilde{\gamma}_0 \tilde{\alpha}}{1 + \tilde{\gamma}_0} \log(u) \right\}}$$

$$\approx \frac{\tilde{\gamma}_0^2}{(1 + \tilde{\gamma}_0)^3} (1 + 2\tilde{\alpha} \log(u)) \left(1 - 3 \frac{\tilde{\gamma}_0 \tilde{\alpha}}{1 + \tilde{\gamma}_0} \log(u) \right)$$

$$\approx \frac{\tilde{\gamma}_0^2}{(1 + \tilde{\gamma}_0)^3} \left(1 + \tilde{\alpha} \log(u) \left[2 - \frac{3\tilde{\gamma}_0}{1 + \tilde{\gamma}_0} \right] \right)$$

⇒

$$\frac{\gamma^2}{(1+\gamma)^3} \approx \frac{\tilde{\gamma}_0^2}{(1+\tilde{\gamma}_0)^3} \left[1 + \left(\frac{2-\tilde{\gamma}_0}{1+\tilde{\gamma}_0} \right) \tilde{\alpha} \log(u) \right]$$

⇒ We can rewrite the differential equation:

$$\frac{\partial \theta}{\partial u} = \left[\frac{\gamma}{1+\gamma} - \left(\frac{u}{k_B T} \right) \frac{\gamma^2}{(1+\gamma)^3} - \theta \right] \cdot \frac{\tilde{V}_0}{\alpha \beta S}$$

⇒

$$\frac{\partial \theta}{\partial u} = \left[\frac{\tilde{\gamma}_0}{1+\tilde{\gamma}_0} - \left(\frac{u}{k_B T} \right) \frac{\tilde{\gamma}_0^2}{(1+\tilde{\gamma}_0)^3} - \theta \right] \frac{\tilde{V}_0}{\alpha \beta S}$$

$$+ \left[\frac{\tilde{\gamma}_0}{(1+\tilde{\gamma}_0)^2} - \left(\frac{u}{k_B T} \right) \frac{\tilde{\gamma}_0^2 (2-\tilde{\gamma}_0)}{(1+\tilde{\gamma}_0)^4} \right] \frac{\tilde{\alpha} \tilde{V}_0}{\alpha \beta S} \cdot \log(u)$$

for simplicity in writing, let us introduce two functions of $\tilde{\gamma}_0$:

$$f_0 = \frac{\tilde{\gamma}_0}{1+\tilde{\gamma}_0} - \left(\frac{u}{k_B T} \right) \frac{\tilde{\gamma}_0^2}{(1+\tilde{\gamma}_0)^3}$$

and:

$$f_1 = \frac{\tilde{\gamma}_0}{(1+\tilde{\gamma}_0)^2} - \left(\frac{w}{k_B T}\right) \frac{\tilde{\gamma}_0^2 (2-\tilde{\gamma}_0)}{(1+\tilde{\gamma}_0)^4}$$

as well as:

$$1/\tau = \tilde{V}_0 / \alpha \phi$$

$\Rightarrow$ the differential equation then becomes:

$$\frac{\partial \theta}{\partial u} = [f_0 - \theta] / \tau + f_1 \frac{\tilde{\alpha}}{\tau} \log(u)$$

using wolfram alpha, we can solve this differential equation and find:

$$\theta(u) = f_1 \tilde{\alpha} e^{-u/\tau} \text{Ei}\left(\frac{u}{\tau}\right) + k_1 e^{-u/\tau} + f_0 - f_1 \tilde{\alpha} \log(u)$$

, where k_1 is an integration constant.

We can now substitute back $u = e^{\alpha \phi}$,
with ϕ the actual electric potential.

$\Rightarrow$

$$\Theta(\phi) = f_1 \tilde{\alpha} e^{-e^{\alpha p \phi} / \tau} \operatorname{Ei}\left(\frac{e^{\alpha p \phi}}{\tau}\right) + k_1 e^{-e^{\alpha p \phi} / \tau} + f_0 - f_1 \alpha' p \phi$$

, where k_1 should be found from a boundary condition, for example a potential at which the coverage is known.

The elliptic integral can cause problems in the (numerical) evaluation of the solution.

$\Rightarrow \operatorname{Ei}(e^{\alpha p \phi} / \tau)$ diverges if $\alpha p \phi / \tau \rightarrow 0$.

$\Rightarrow$ we can rewrite the equation back into the time domain:

$$\phi = \phi_0 + s \cdot t$$

$\Rightarrow$

$$\Theta(t) = f_1 \tilde{\alpha} e^{-e^{\alpha p (\phi_0 + s t)} / \tau} \operatorname{Ei}\left(\frac{e^{\alpha p (\phi_0 + s t)}}{\tau}\right) - e^{\alpha p (\phi_0 + s t)} / \tau$$

$$+ k_1 e$$

$$+ f_0 - f_1 \alpha' \beta (\phi_0 + st)$$

$\theta(t=0)$ should be the equilibrium coverage at ϕ_0 , hence:

$$\theta(t=0) = f_1 \tilde{\alpha} \bar{e}^{-e^{\alpha\beta\phi_0}/\tau} \text{Ei}\left(\frac{e^{\alpha\beta\phi_0}}{\tau}\right) + k_1 \bar{e}^{-e^{\alpha\beta\phi_0}/\tau} + f_0 - f_1 \alpha' \beta \phi_0$$

$$\equiv \underbrace{\gamma/(1+\gamma) - (w/a_B T) \gamma^2/(1+\gamma)^3}_{\text{define: } f_2(\phi_0)} \quad [\text{@ } \phi_0]$$

$\Rightarrow$

$$f_1 \tilde{\alpha} \bar{e}^{-e^{\alpha\beta\phi_0}/\tau} \text{Ei}\left(\frac{e^{\alpha\beta\phi_0}}{\tau}\right) + k_1 \bar{e}^{-e^{\alpha\beta\phi_0}/\tau} + f_0 - f_1 \alpha' \beta \phi_0 = f_2(\phi_0)$$

$$\Rightarrow k_1 = \left[f_2(\phi_0) - f_0 + f_1 \alpha' \beta \phi_0 \right] \bar{e}^{e^{\alpha\beta\phi_0}/\tau} - f_1 \tilde{\alpha} \text{Ei}\left(\frac{e^{\alpha\beta\phi_0}}{\tau}\right)$$

we do have a solution...:

, which results in the following final solution for $\Theta(\phi)$:

Till now
not capable of plotting....

$$+ k_1 e^{-e^{\alpha\beta\phi}/\tau} + f_0 - f_1 \alpha' \beta \phi$$

$$\Theta(\phi) = f_1 \tilde{\alpha} \left[\text{Ei}\left(\frac{e^{\alpha\beta\phi}}{\tau}\right) - \text{Ei}\left(\frac{e^{\alpha\beta\phi_0}}{\tau}\right) \right] e^{-e^{\alpha\beta\phi}/\tau} \\ + \left[f_2(\phi_0) - f_0 + f_1 \alpha' \beta \phi_0 \right] e^{\left[e^{\alpha\beta\phi_0} - e^{\alpha\beta\phi} \right] / \tau} \\ + f_0 - f_1 \alpha' \beta \phi_0$$

, which is only valid for $\phi \geq \phi_0$!

=

ϕ is the potential you want to know Θ

ϕ_0 is the starting potential

S is the sweep rate of the potential

$\beta = 1/k_B T$, k_B Boltzmann's constant, and T temperature

$\tilde{\gamma}_0 = e^{-E_{\text{form}}/\tau}$ where E_{form} is the formation energy

$\tilde{v}_0 = v_0 e^{-\beta E_d}$, where v_0 is the attempt frequency and E_d is the diffusion energy.

α = correction to diffusion energy with potential

α' = correction to formation energy with potential

Question:

Unless you do temperature dependent measurements, I do not think v_0 and the diffusion energy E_d are independent fitting parameters, as they come together in:

$$\tilde{v}_0 = v_0 \exp(-\beta E_d) \quad \square$$

Acknowledgments:



B. Valbæk Mygind



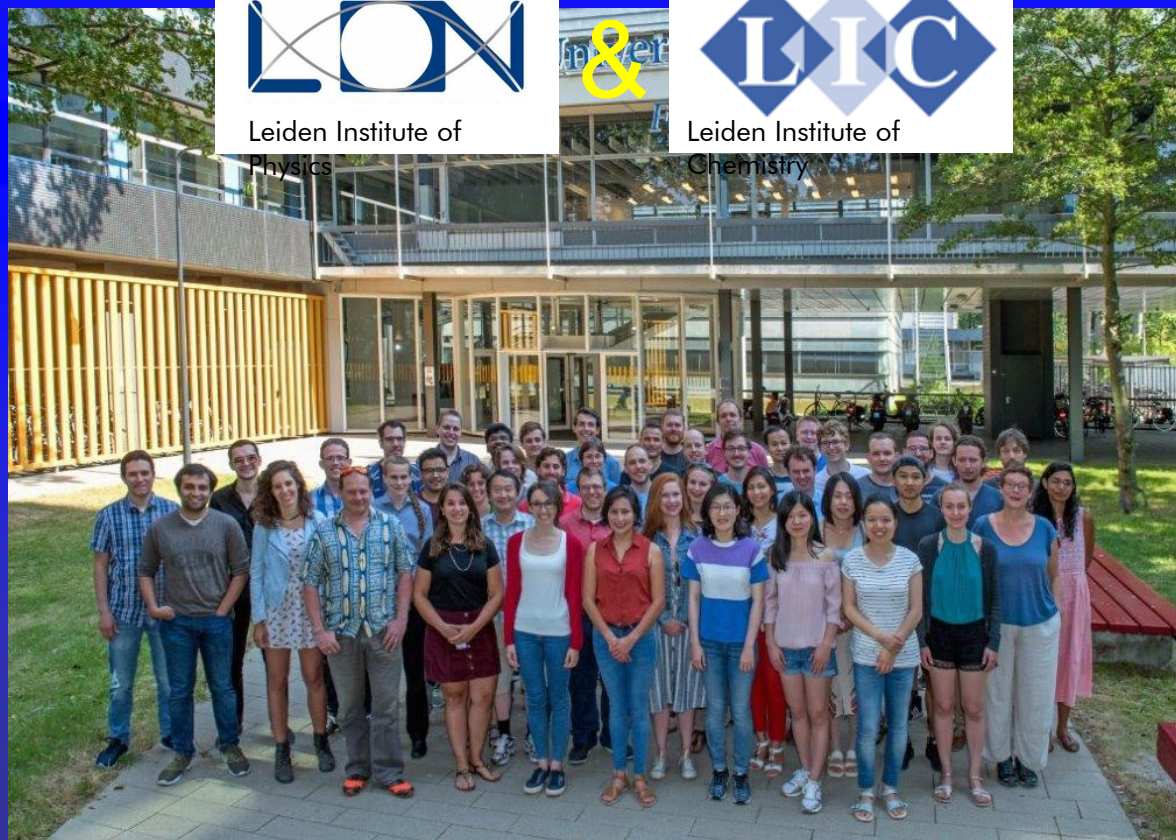
F. Valls Mascaro



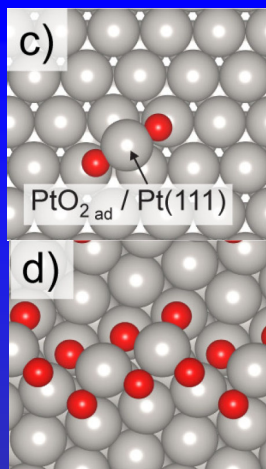
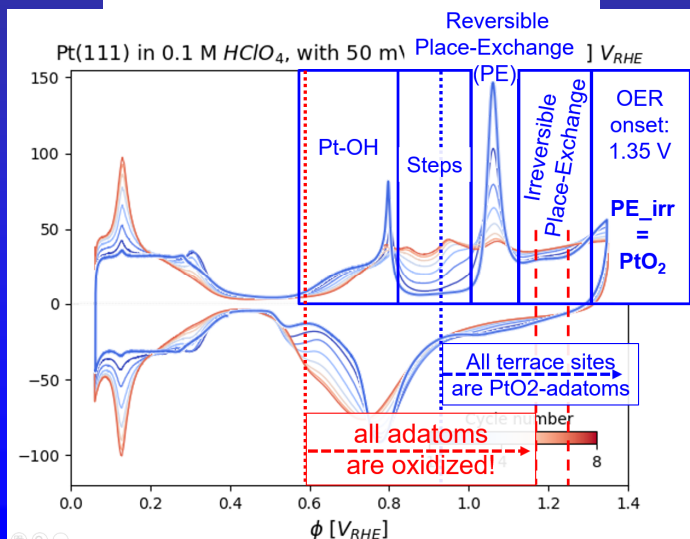
Prof. G. Verbiest



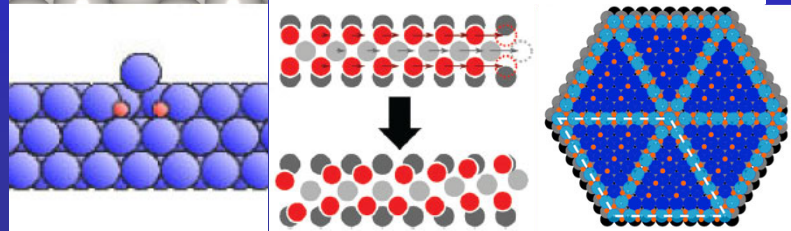
Prof. M. Koper



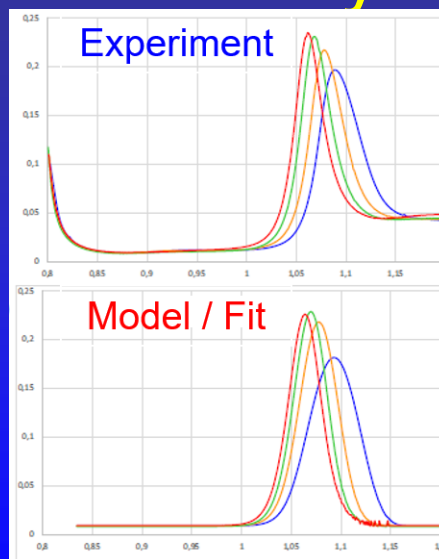
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Summary:



Best Fit:

$$E^0_{\text{form}} = 1.11 \text{ eV} \ \& \ \lambda_{\text{form}} = 0.0704$$

$$w = 0.0068$$

$$v_0 = 6.9\text{E}13$$

$$E^0_{\text{diff}} = 1.27 \text{ eV} \ \& \ \lambda_{\text{diff}} = 0.7278$$

