



A snow isothermal metamorphism model applicable on microtomographic images

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Elie Bretin, Roland Denis, Jacques-Olivier Lachaud and Edouard Oudet

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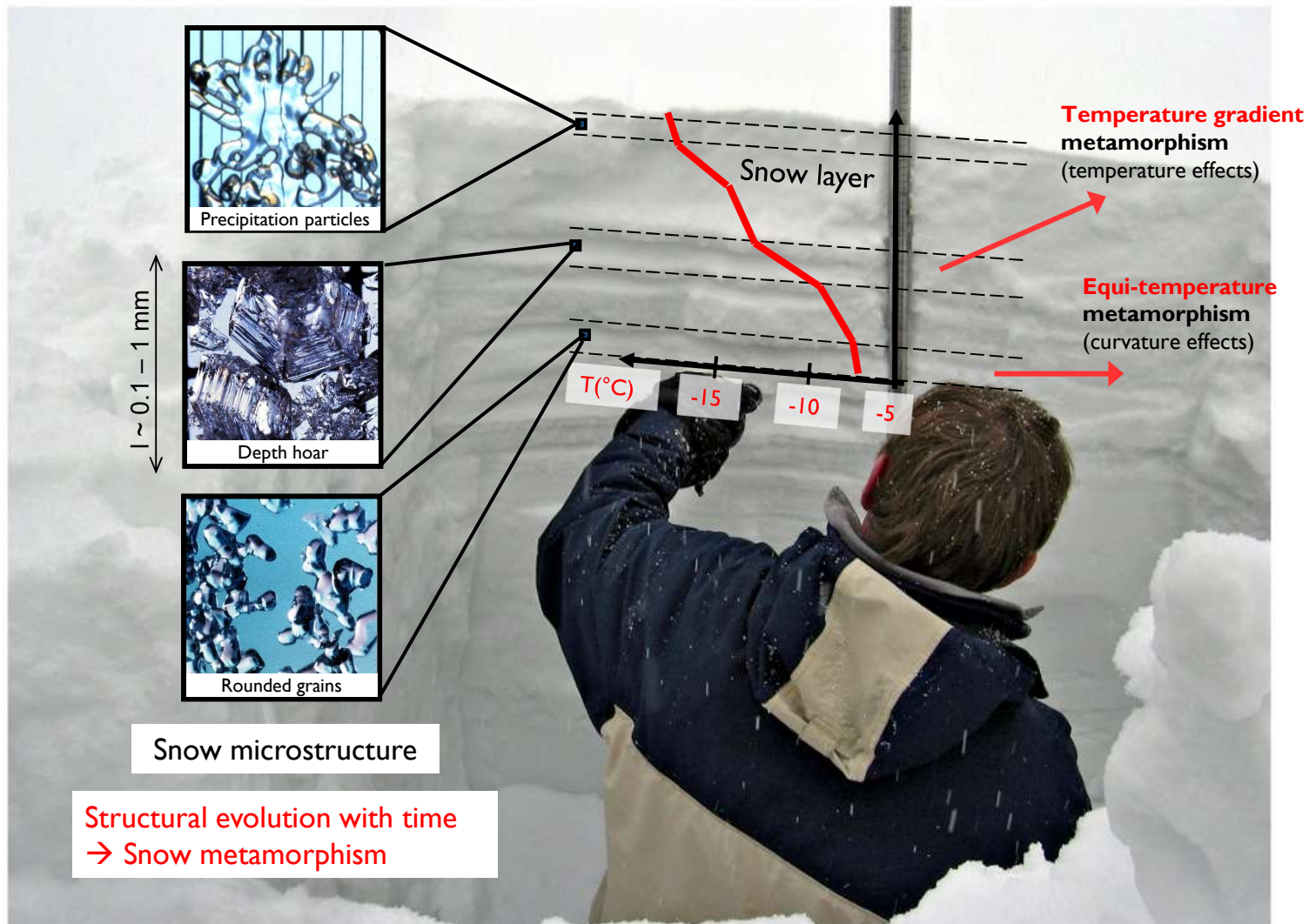
Context – importance of a good knowledge of the snowpack

Societal issues

- Avalanche risk forecasting
- Climate studies
- Water resources management...



Context – snowpack properties are changing with time

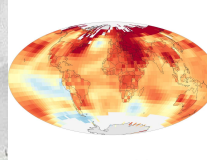


Context – need of microscale observations

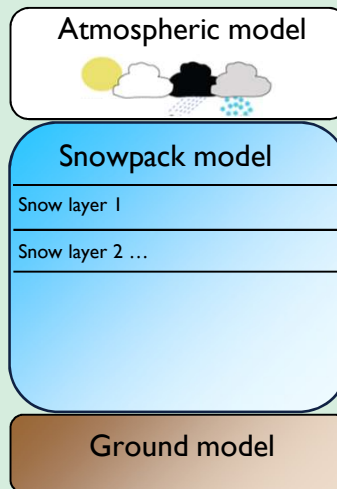
Societal issues

- Avalanche risk forecasting
- Climate studies
- Water resources...

RECENT TEMPERATURE TRENDS (1990-2021)



Snowpack models for the prevision of the time evolution of the macroscopic properties (temperature, mechanical properties...)

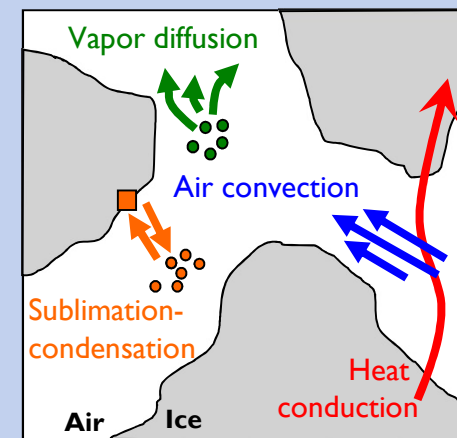


- Physical processes
 - Heat conduction
 - Vapor diffusion
 - Air convection
 - Phase change (source terms)
 - Compaction
 - ...
- Effective properties = $f(\text{microstructure})$

MACROSCALE (Snowpack)

Snow metamorphism

- Heat and mass transfer



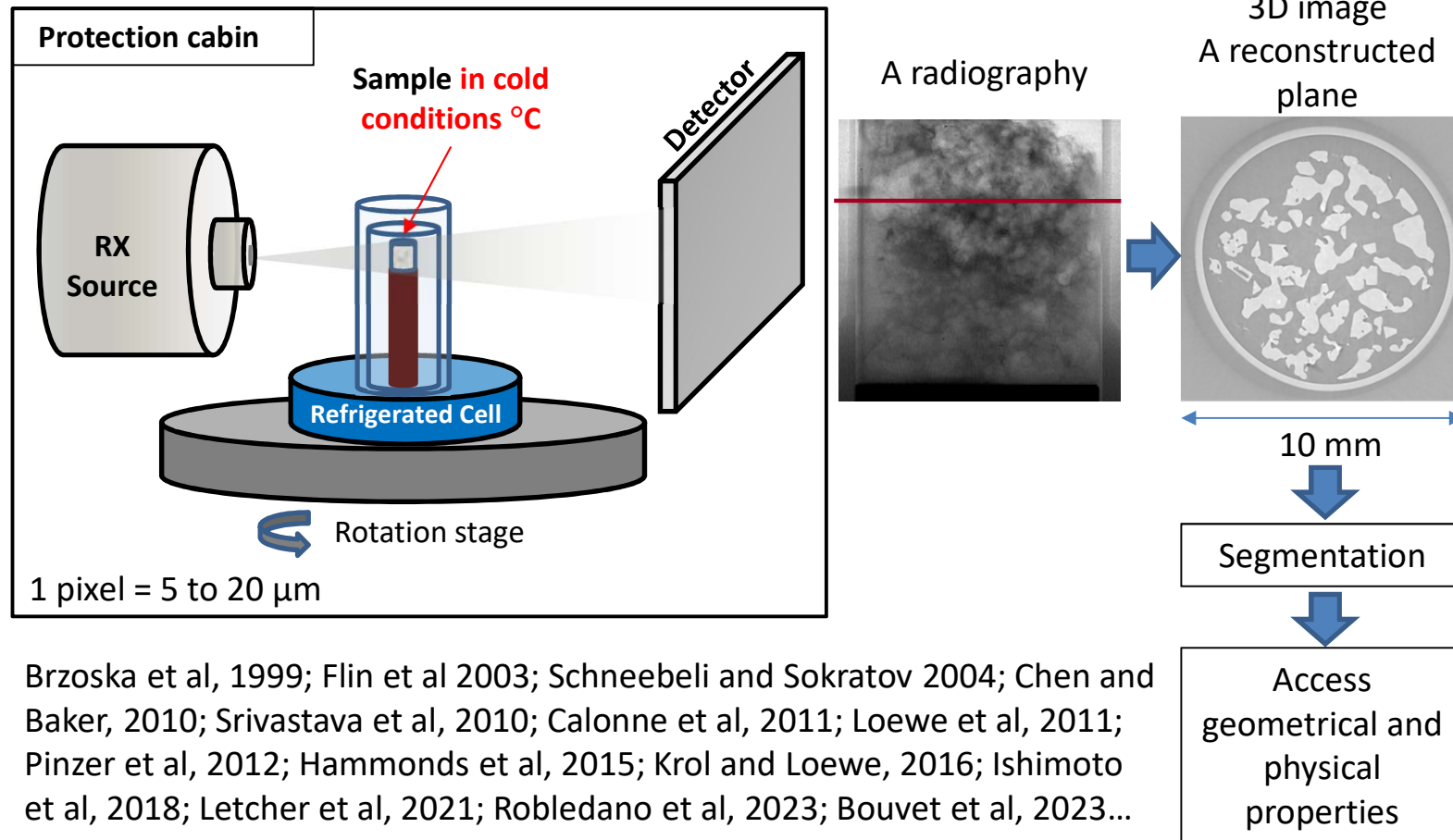
- Ice crystal growth



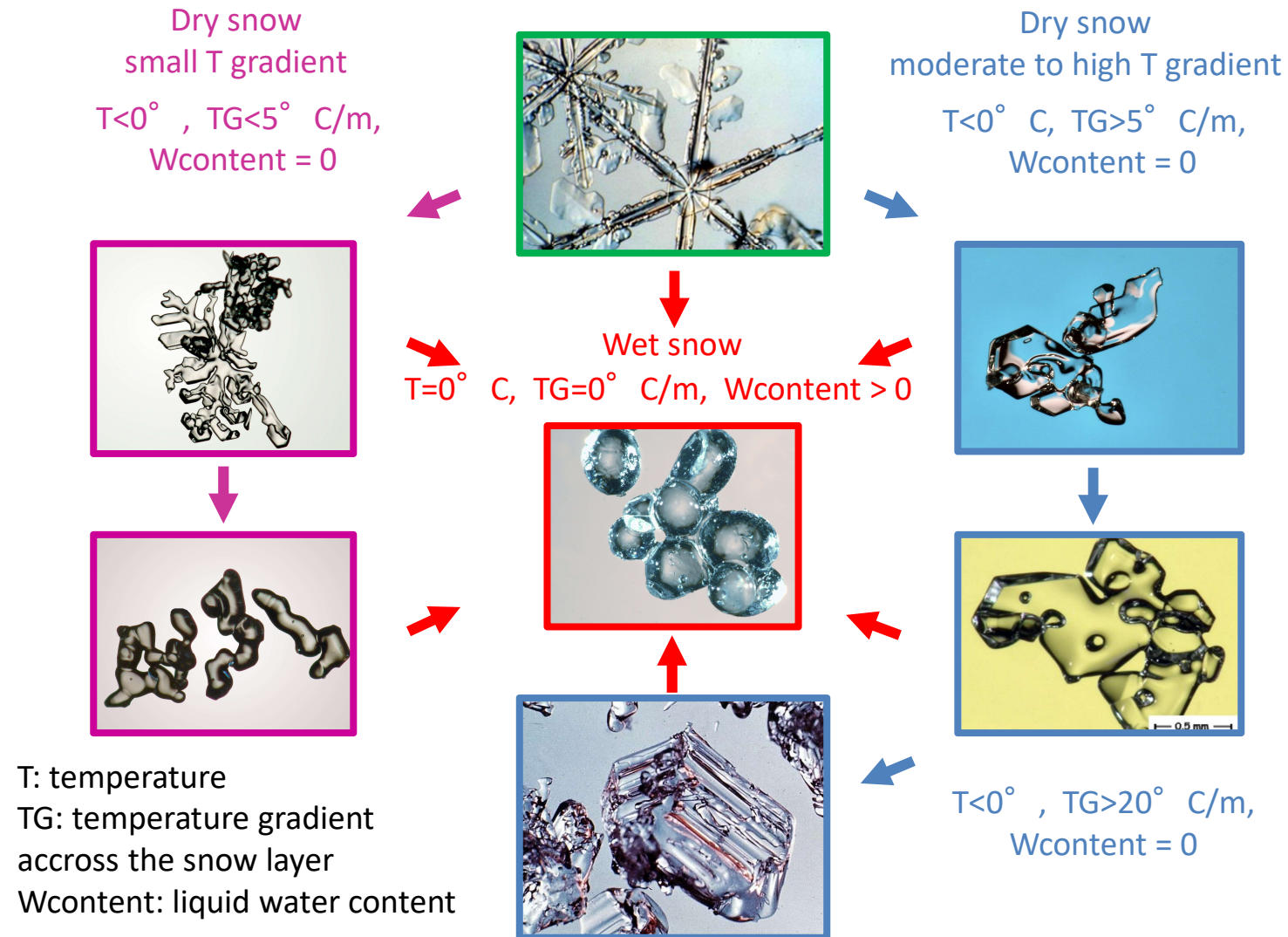
MICROSCALE (Grain assembly)

→ Needs fine observations

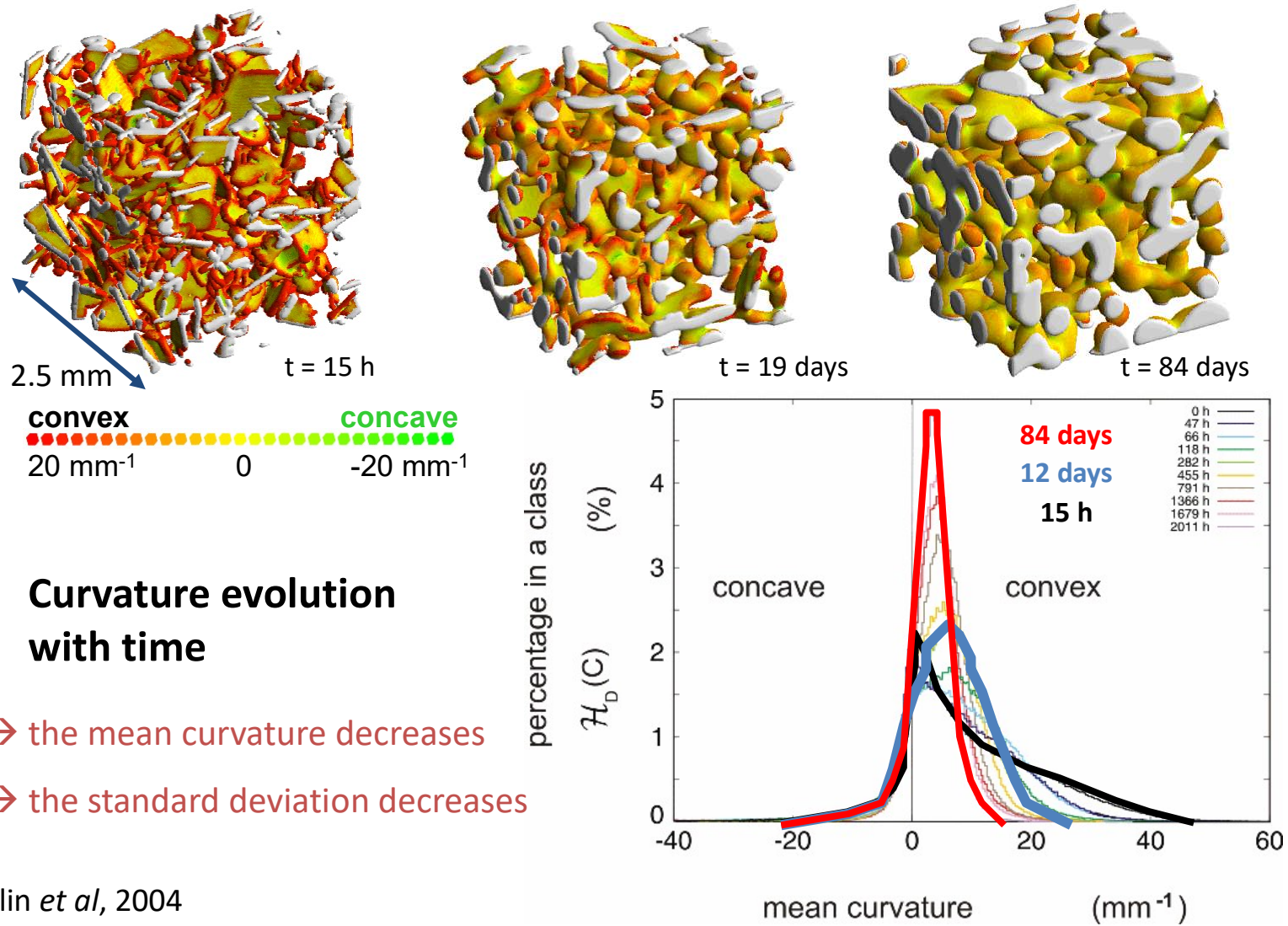
Context – X-ray microtomography to obtain 3D images



Context – Several types of grains and metamorphism regimes



Context – Isothermal metamorphism



Equations – Physical modelling

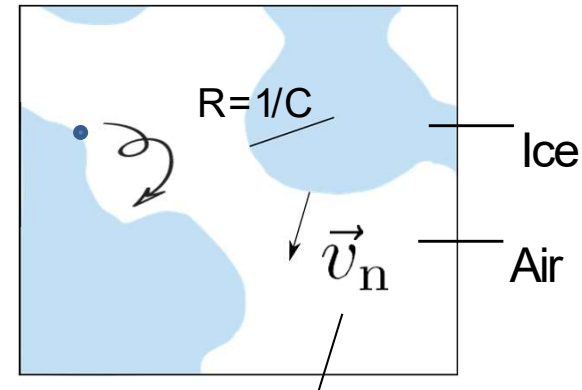
Sublimation-deposition process:

without diffusion, convection and settling of grains

Mechanism: Hertz-Knudsen equation

$$v_n = \alpha v_{\text{kin}} \frac{\rho_{vs}^{\text{amb}} - \rho_{vs}^{\Gamma}}{\rho_{vs}^{\Gamma}} \quad \text{on } \Gamma$$

$$\text{with } v_{\text{kin}} = \frac{\rho_{vs}^{\text{ref}}}{\rho_i} \sqrt{\frac{kT}{2\pi m}}$$



ρ_{vs}^{Γ} : saturation vapor density on the ice-air interface

Deposition coefficient α : probability that a water vapor molecule is incorporated into the ice phase.
Today still poorly quantified!

Belongs to $[0, 1]$ and depends at least on temperature T .

Driving force: Kelvin's (or Gibbs-Thomson) law

$$\rho_{vs}^{\Gamma} = \rho_{vs}^{\text{ref}} e^{2d_0 C} \quad \text{on } \Gamma$$

$$\rho_{vs}^{\text{amb}} = \rho_{vs}^{\text{ref}} e^{2d_0 C^{\text{amb}}}$$

C : local curvature

C_{amb} : integrated curvature on whole interface

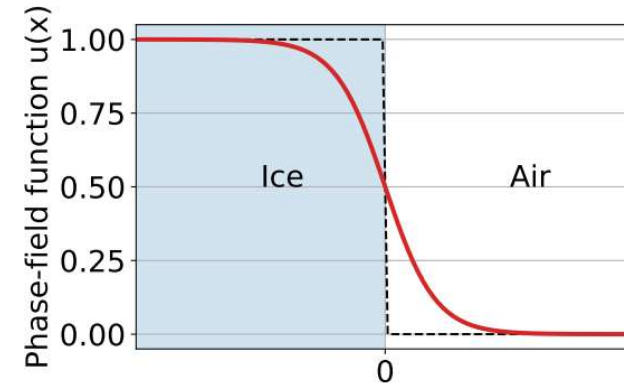
d_0 : capillary length $\ll 1$

→ $v_n = 2d_0 \alpha v_{\text{kin}} (C_{\text{amb}} - C)$ Typical case of volume preserving curvature flow

Equations – Phase-field modelling: Bretin et al (2019)

Application of classical phase-field equation to the case of isothermal metamorphism:

$$\frac{\partial u}{\partial t}(x, t) = d_0 \alpha v_{\text{kin}} \left(\Delta u(x, t) - \frac{1}{\varepsilon^2} W'(u) \right)$$



Following Bretin et al (2019), the canonical dimensionless equation can be written:

$$\frac{\partial \tilde{u}}{\partial \tilde{t}}(\tilde{x}, \tilde{t}) = \Delta \tilde{u}(\tilde{x}, \tilde{t}) - \frac{1}{\tilde{\varepsilon}^2} W' + \tilde{\mu} \frac{1}{\tilde{\varepsilon}} \sqrt{2W}(\tilde{u}, \tilde{t})$$

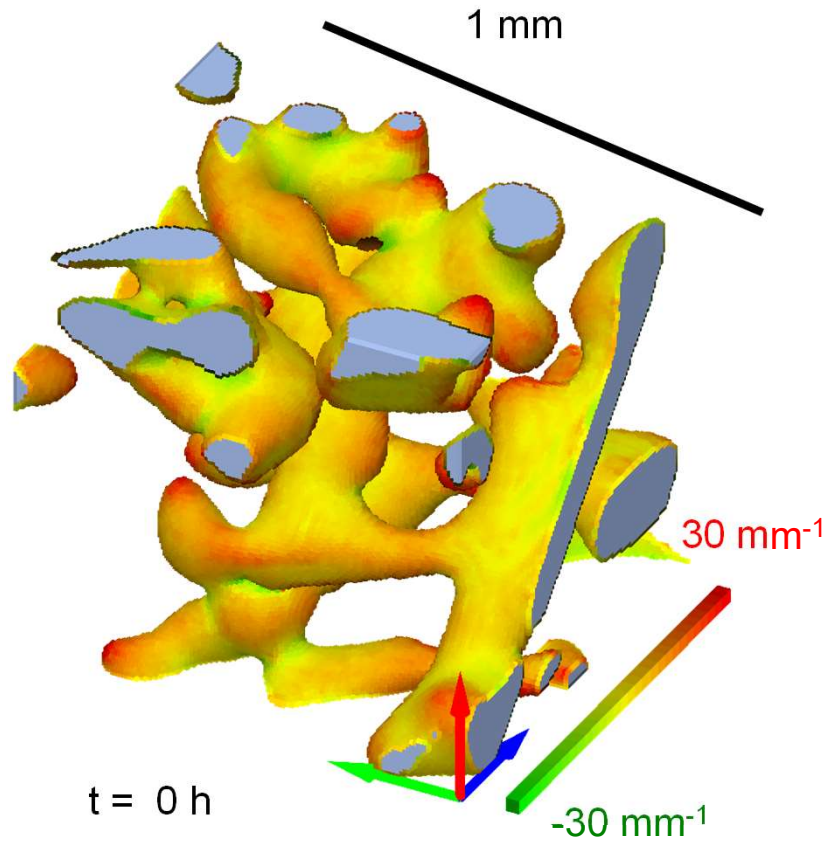
Where: $\tilde{t} = \frac{t \alpha v_{\text{kin}} d_0}{d_x^2}$, $\tilde{x} = \frac{x}{d_x}$, $\tilde{\varepsilon} = \frac{\varepsilon}{d_x}$

And $\tilde{\mu}$ is a Lagrange multiplier verifying:
$$\tilde{\mu} = \frac{1}{\tilde{\varepsilon}} \frac{\int_{\Omega} W'(\tilde{u}, \tilde{t}) d\tilde{x}}{\int_{\Omega} \sqrt{2W}(\tilde{u}, \tilde{t}) d\tilde{x}}$$

Bretin et al, 2019: Phase-field modelling and computing for a large number of phases,
Elie Bretin, Roland Denis, Jacques-Olivier Lachaud and Edouard Oudet, ESAIM: M2AN, 53
3 (2019) 805-832, DOI: <https://doi.org/10.1051/m2an/2018075>

Comparison between experiments and simulation

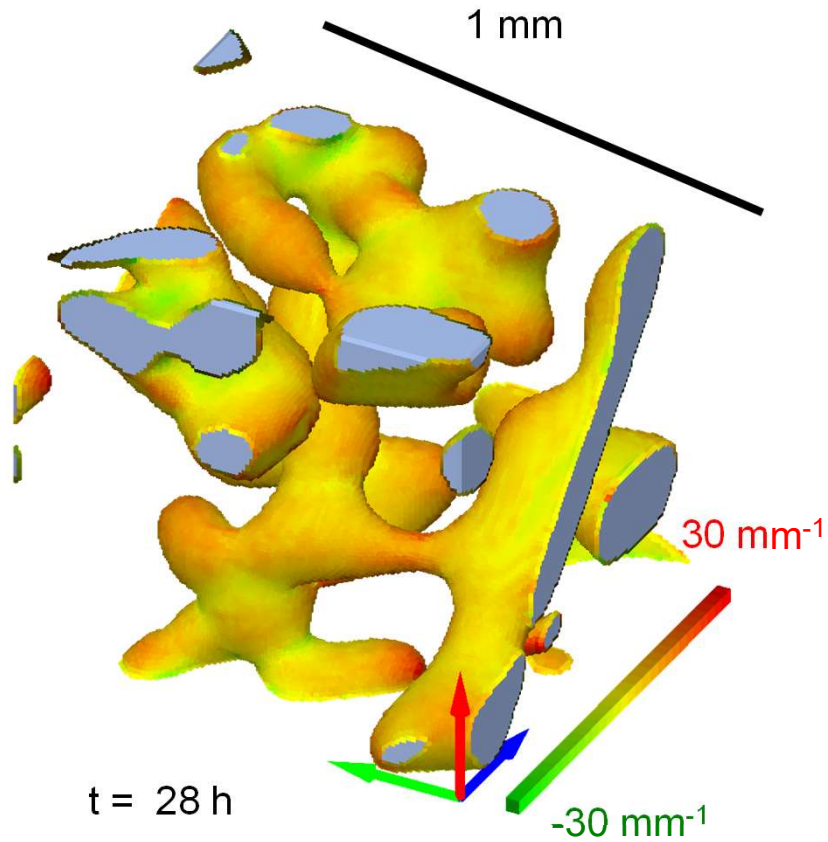
28 h of equi-temperature experiment at -7°C



Experimental result

Comparison between experiments and simulation

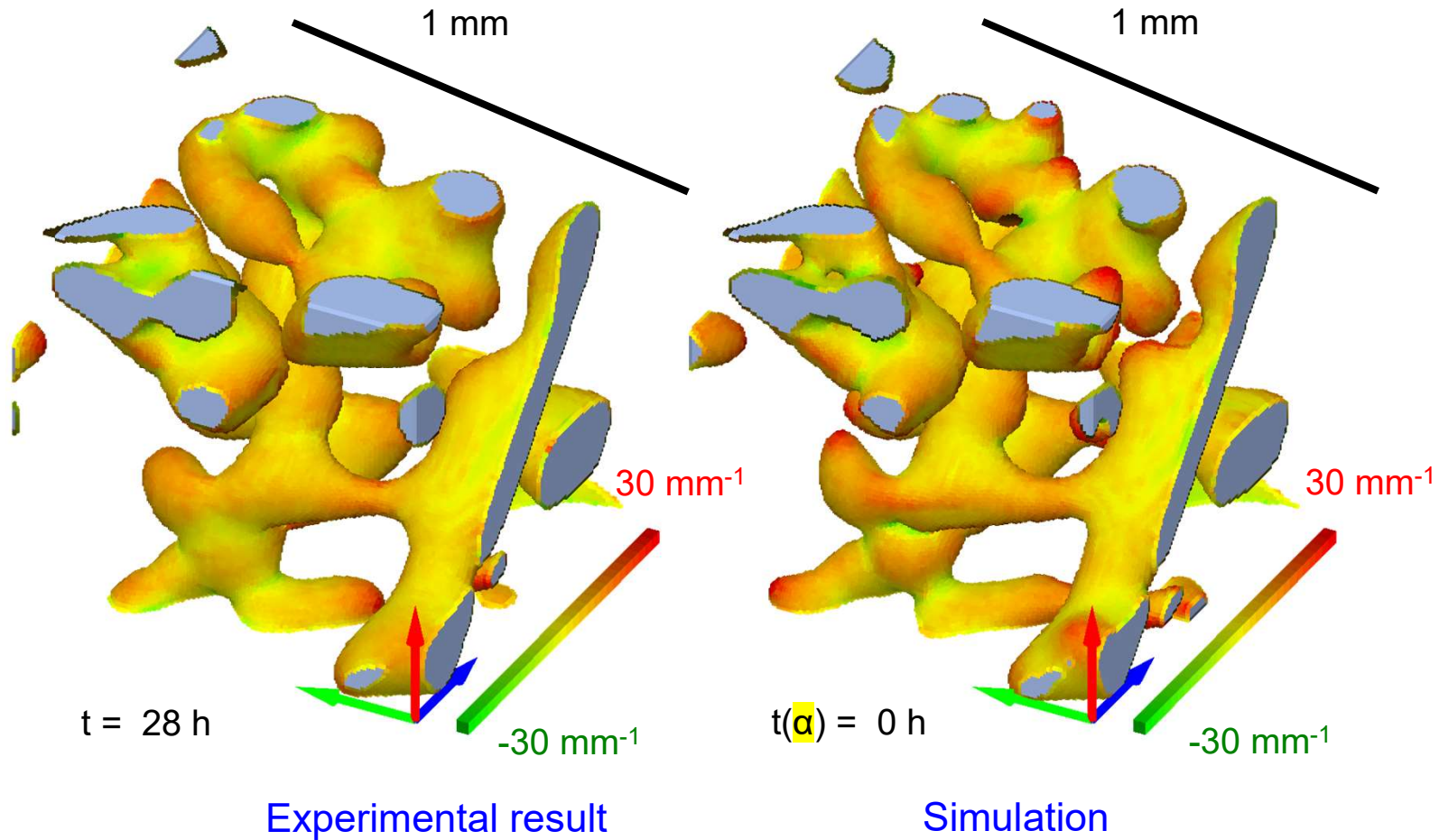
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Experimental result

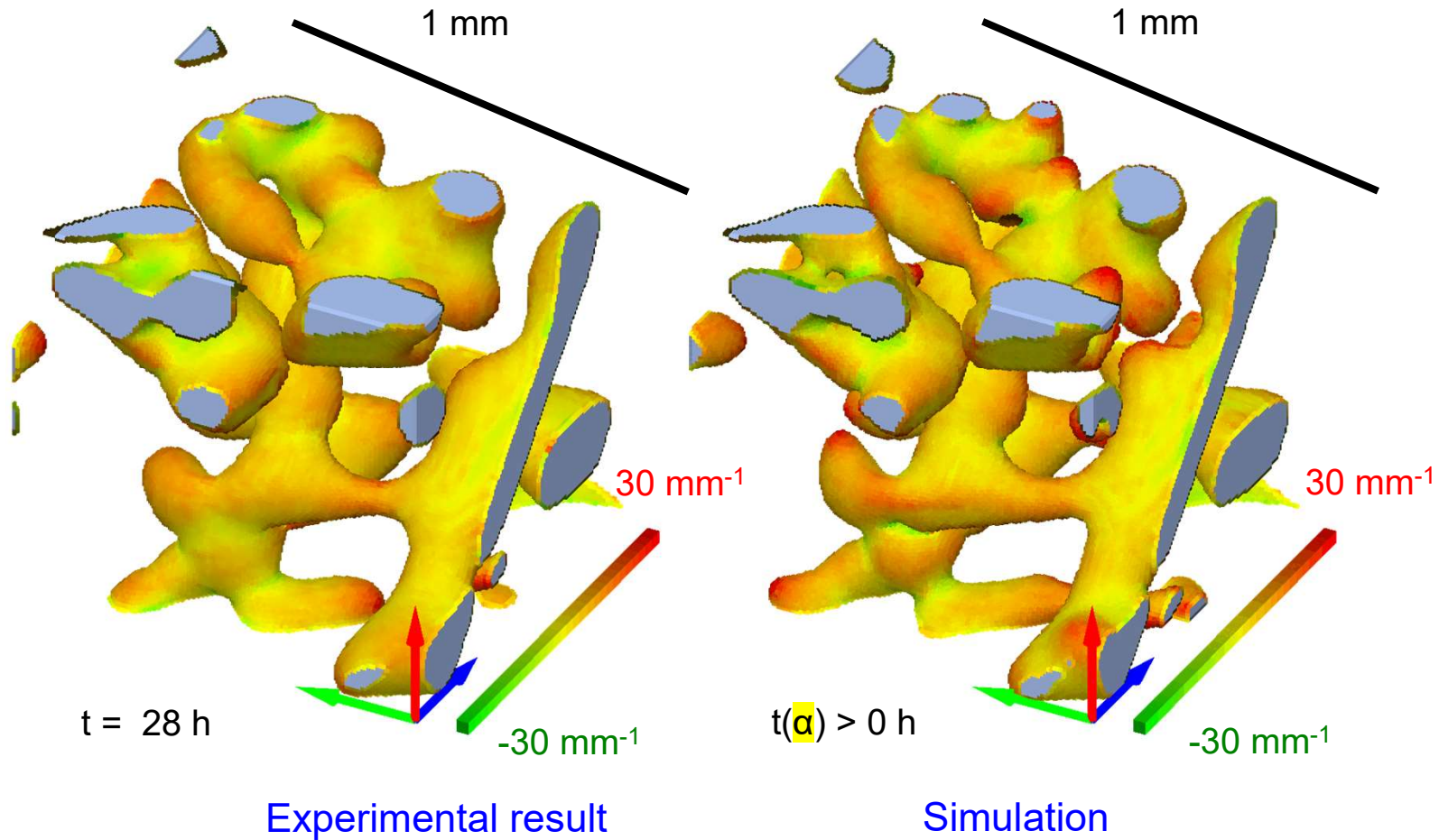
Comparison between experiments and simulation

28 h of equi-temperature experiment at -7°C



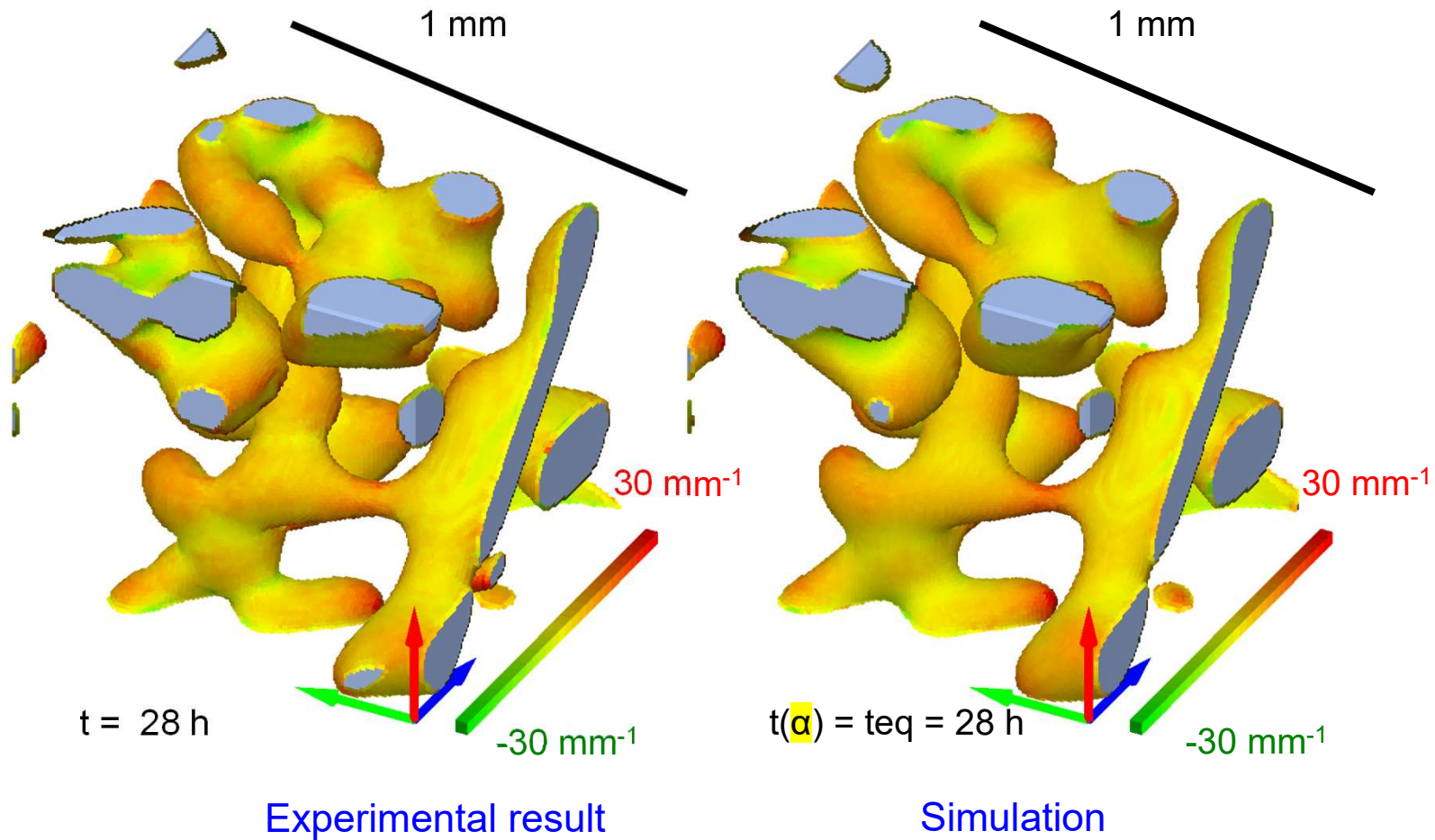
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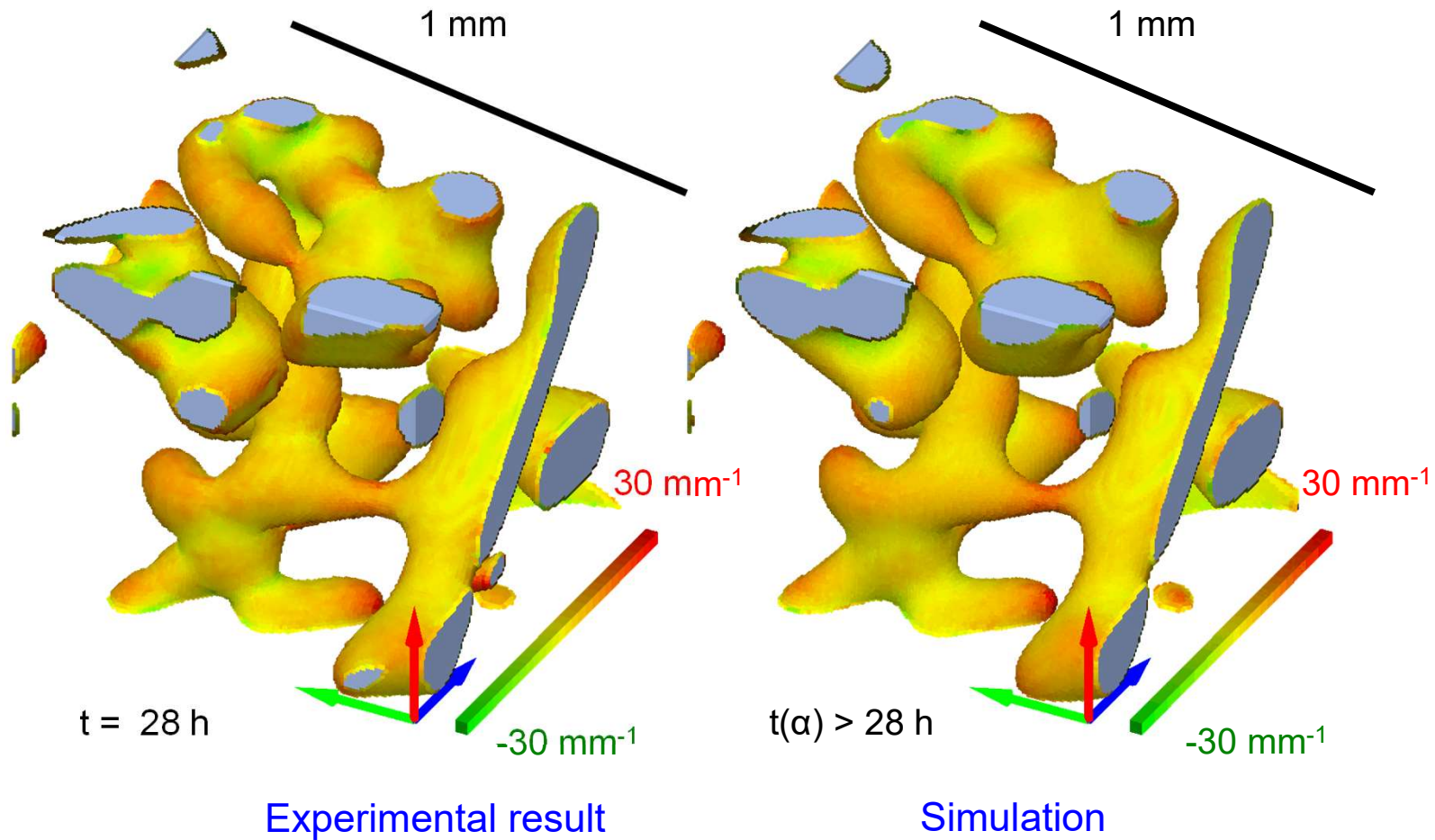
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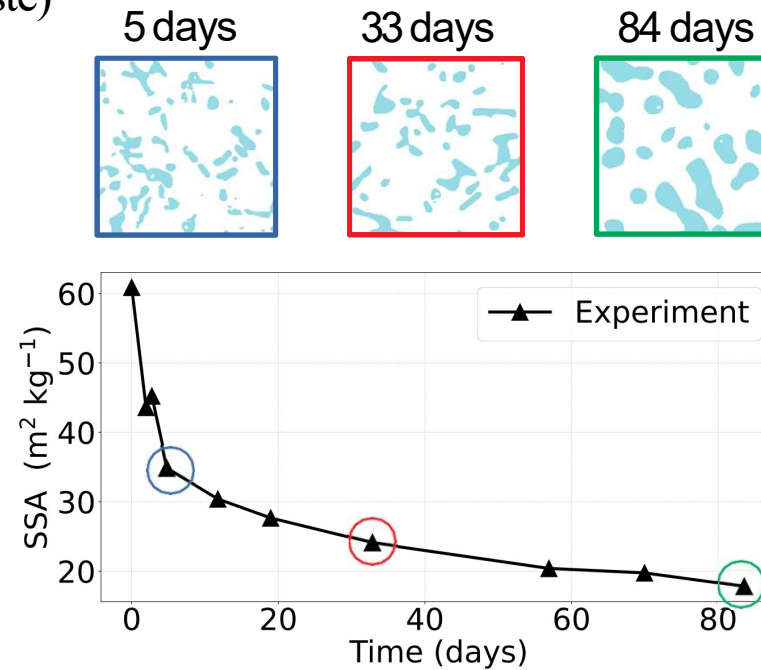
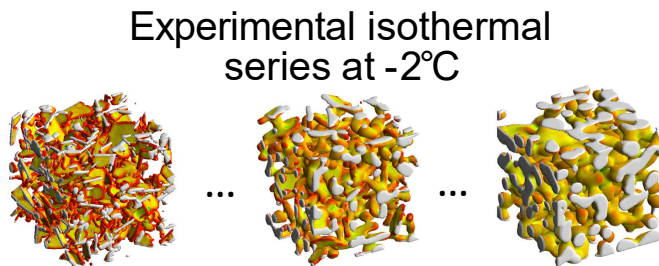
Model calibration:

Link between the non-dimensional time $\tilde{t}$ and the physical time t :

$$t = \frac{\tilde{t} d_x^2}{\alpha v_{\text{kin}} d_0}$$

$\tilde{t}$: image resolution (cste)
 d_x : image resolution (cste)
 α : condensation coefficient
 v_{kin} : kinetic velocity (cste)
 d_0 : capillary length (cte)

condensation coefficient



Model calibration:

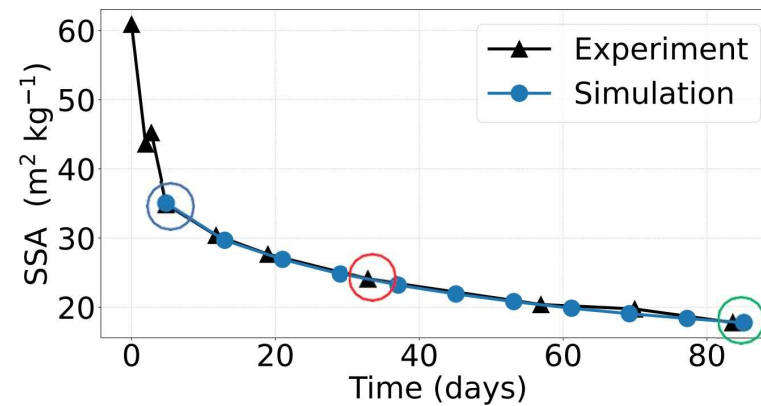
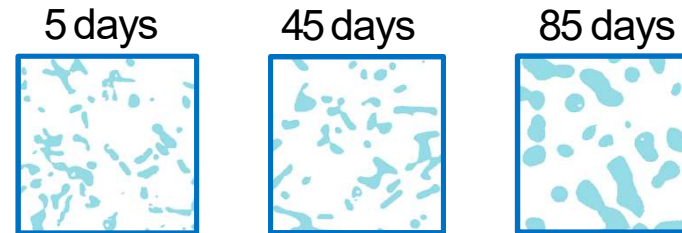
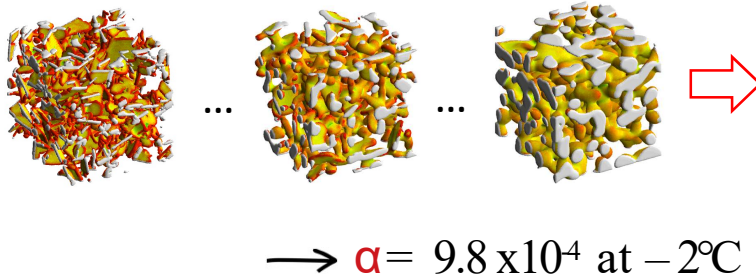
Link between the non-dimensional time $\tilde{t}$ and the physical time t :

$$t = \frac{\tilde{t} d_x^2}{\alpha v_{\text{kin}} d_0}$$

image resolution (cte)
 kinetic velocity (cte)
 capillary length (cte)
 kinetic velocity (cte)

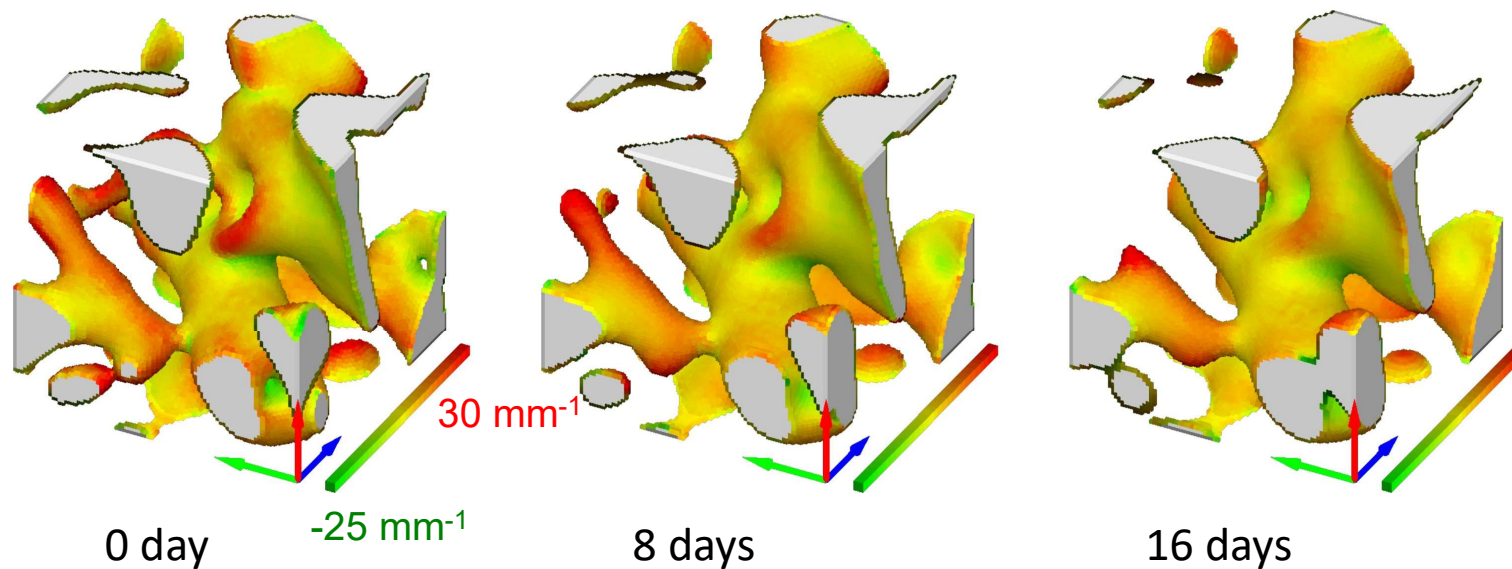
condensation coefficient

Simulated series



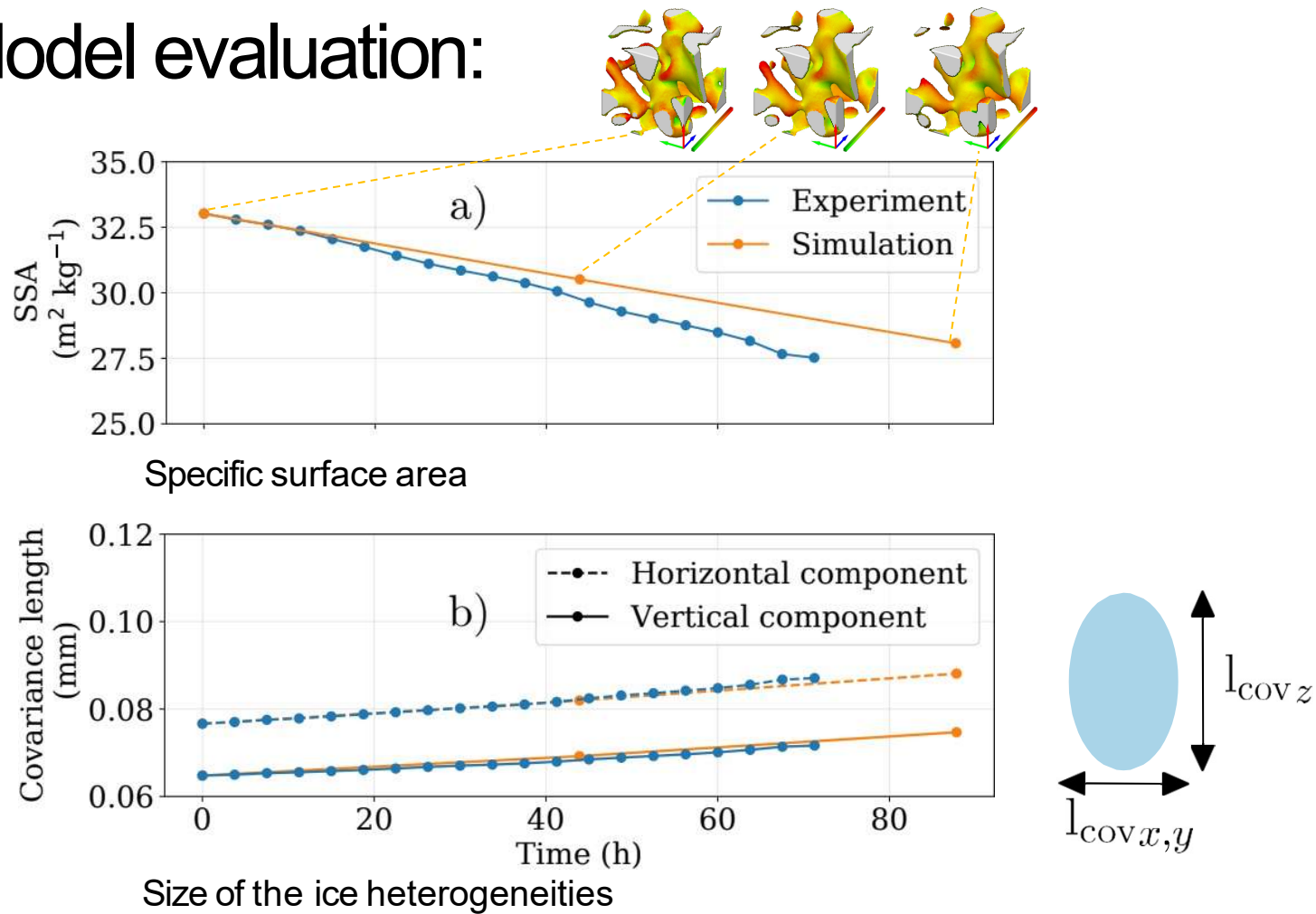
Model evaluation:

Evaluation of α using an independent experimental series of isothermal metamorphism at -2°C (Hagenmuller et al, 2019).



Simulation of isothermal metamorphism at -2°C .

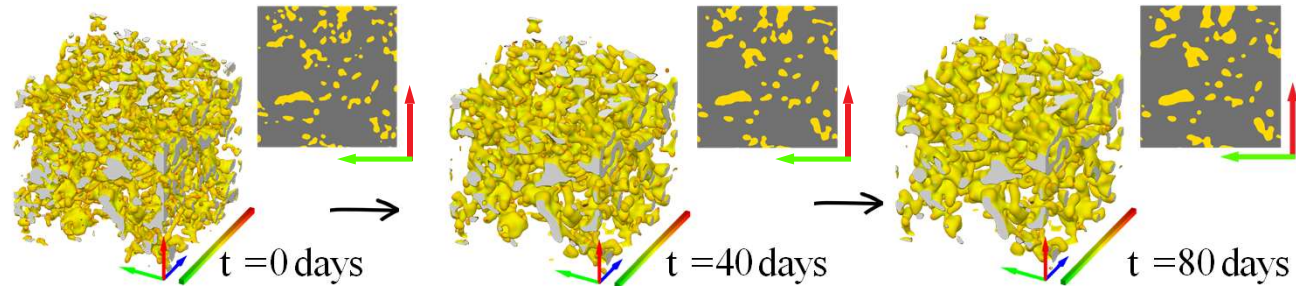
Model evaluation:



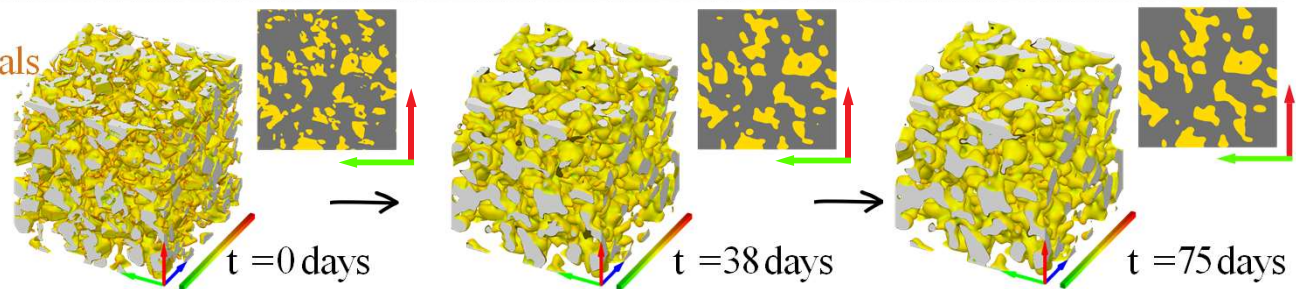
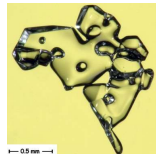
➔ Good agreement between simulated and experimental microstructural parameters.

Model predictions: snow microstructures at -2°C

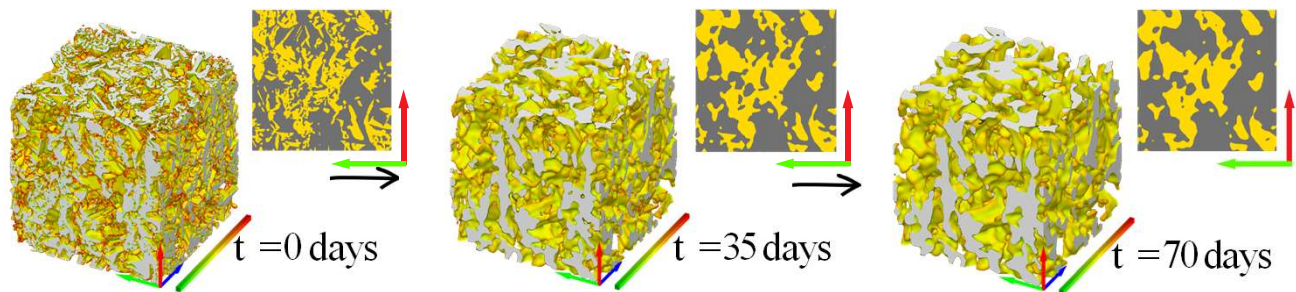
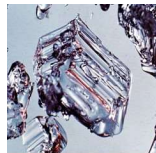
Fresh snow

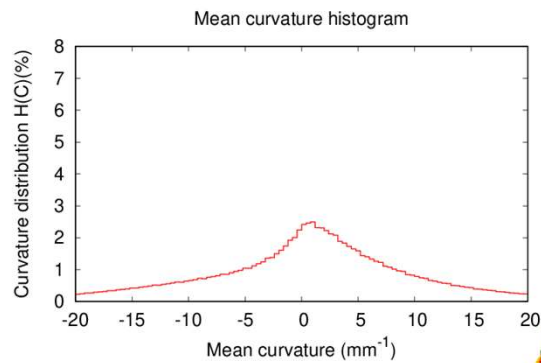


Faceted Crystals

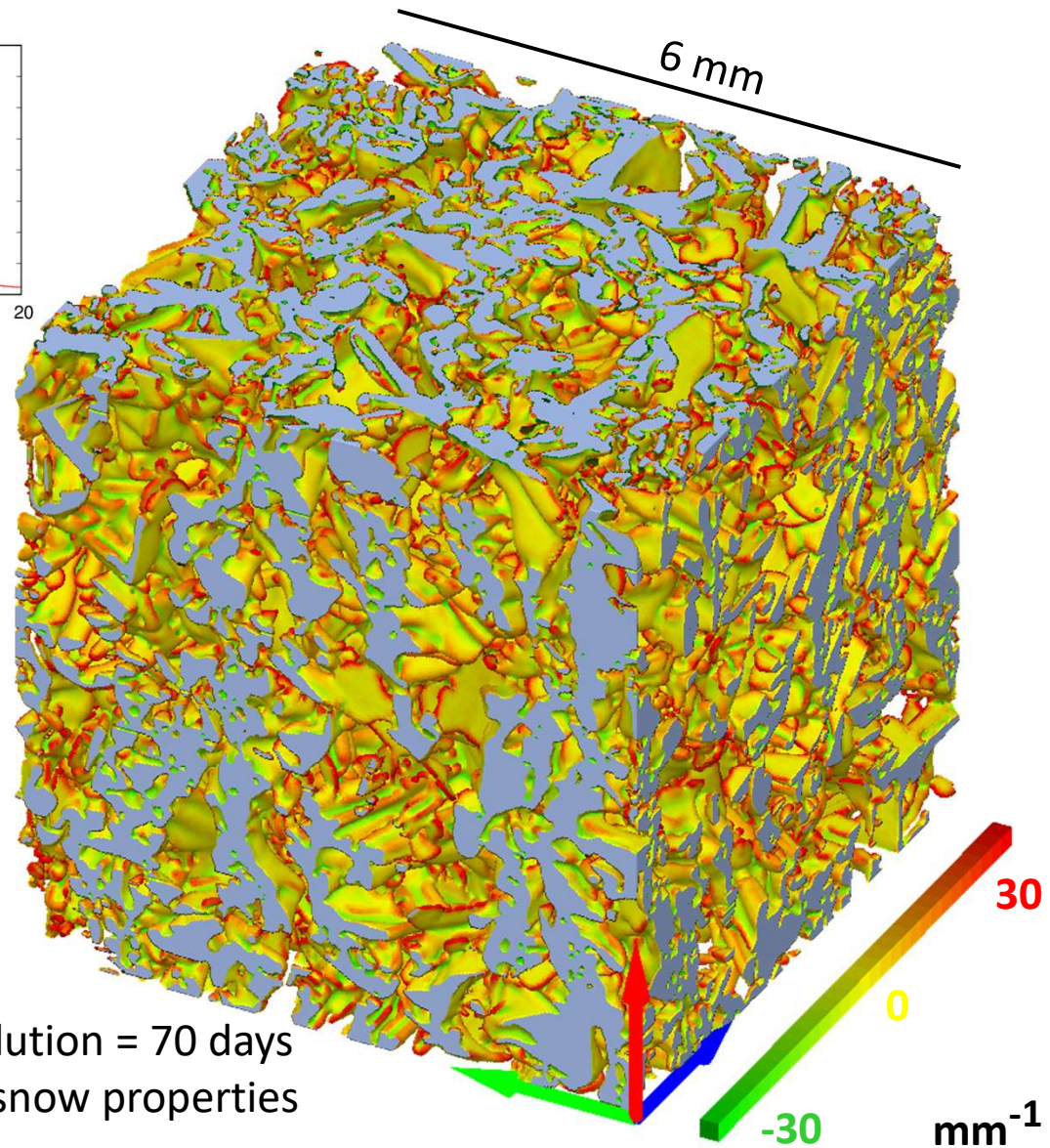


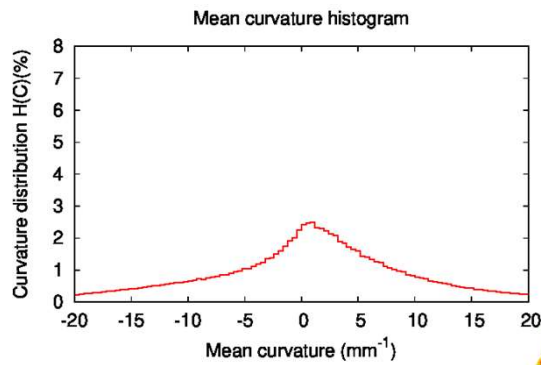
Depth Hoar



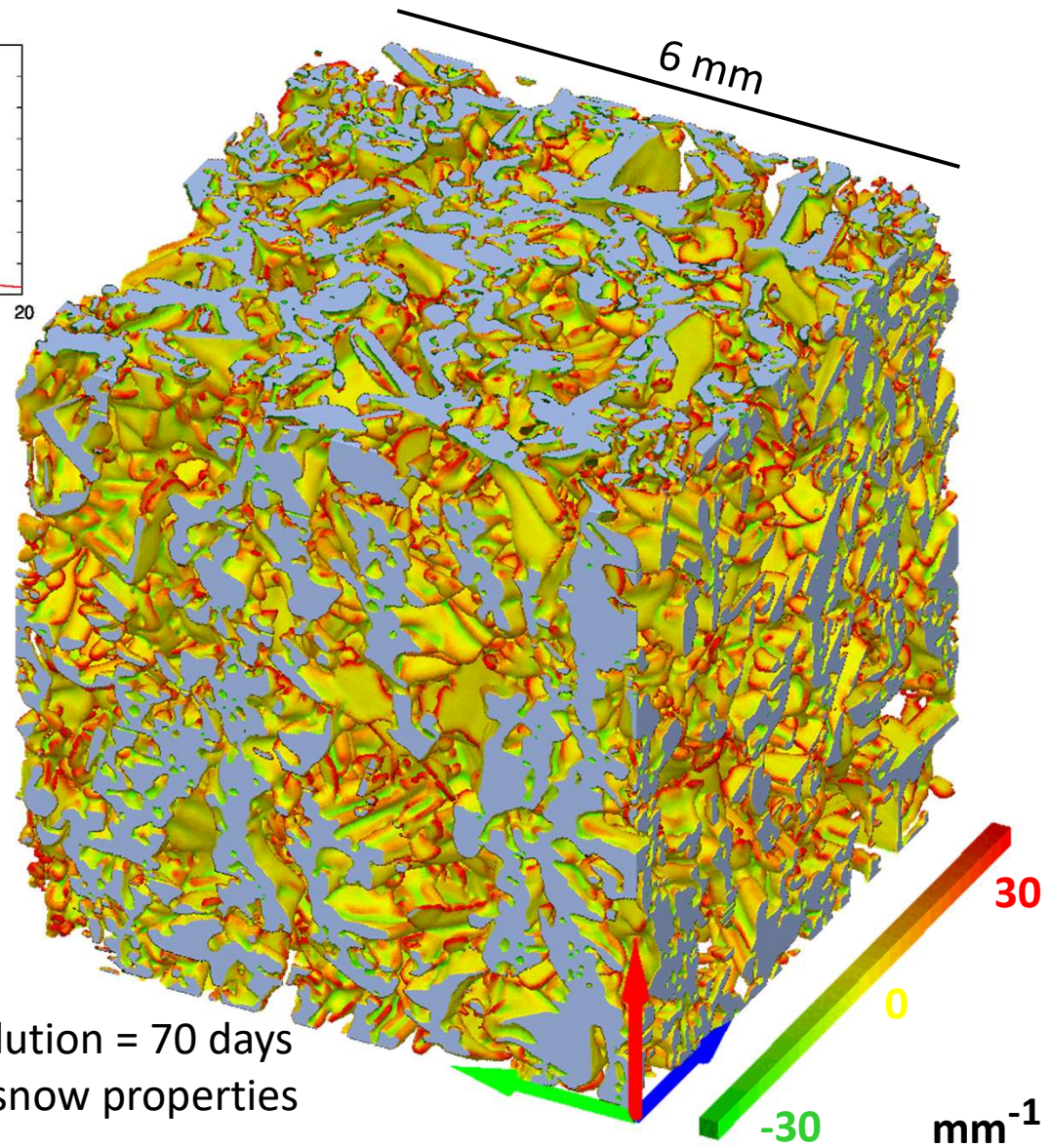


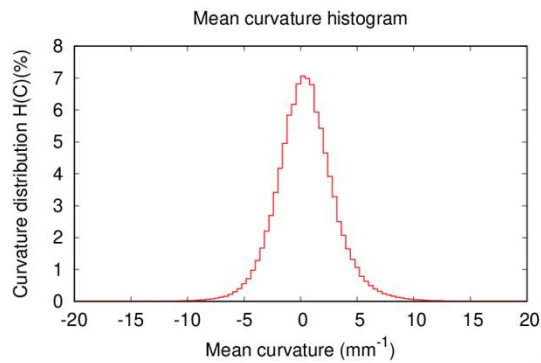
Simulation
of depth hoar
under isothermal
metamorphism at -2°C
Physical time of the evolution = 70 days
➔ Access modelling of snow properties



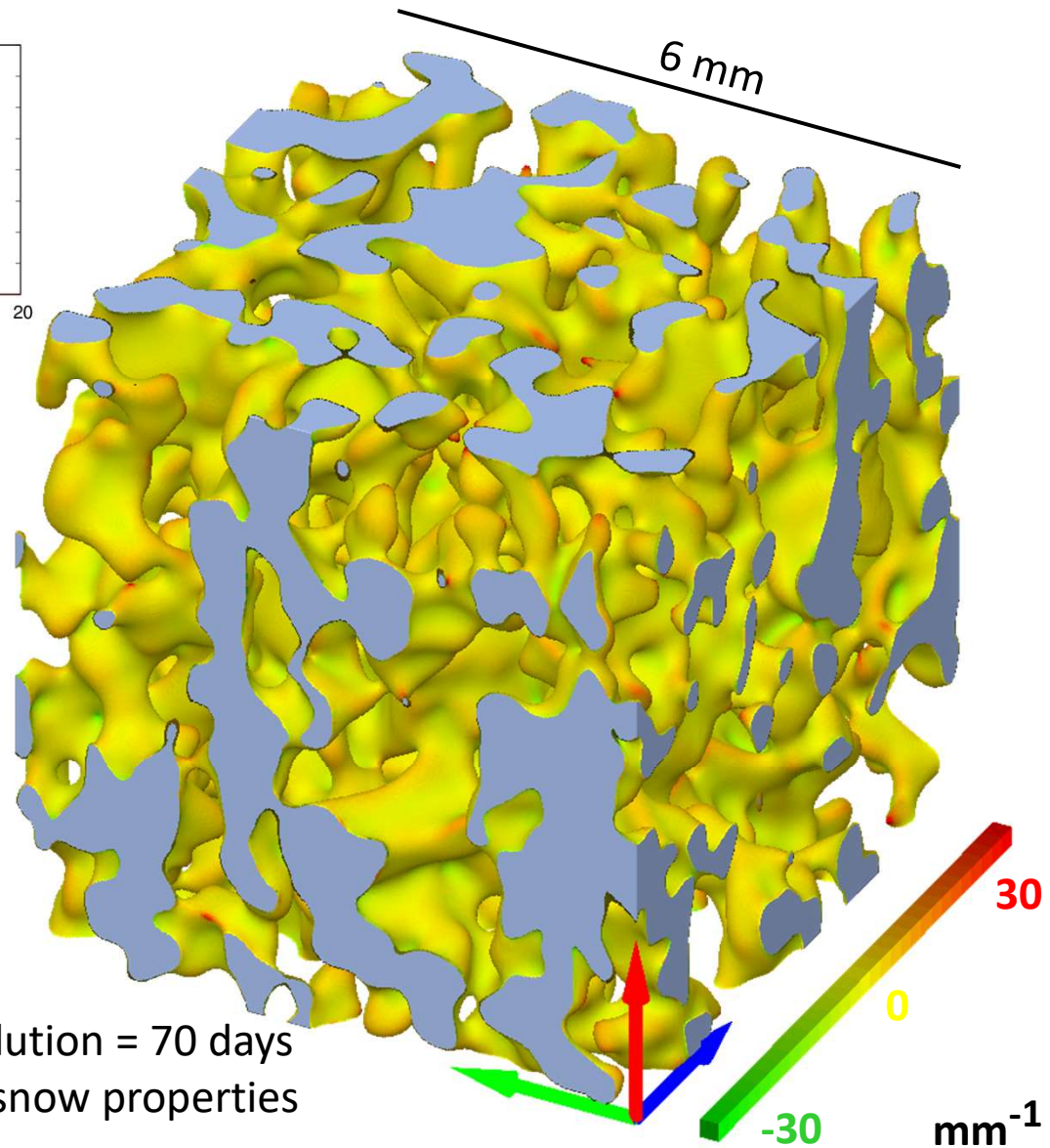


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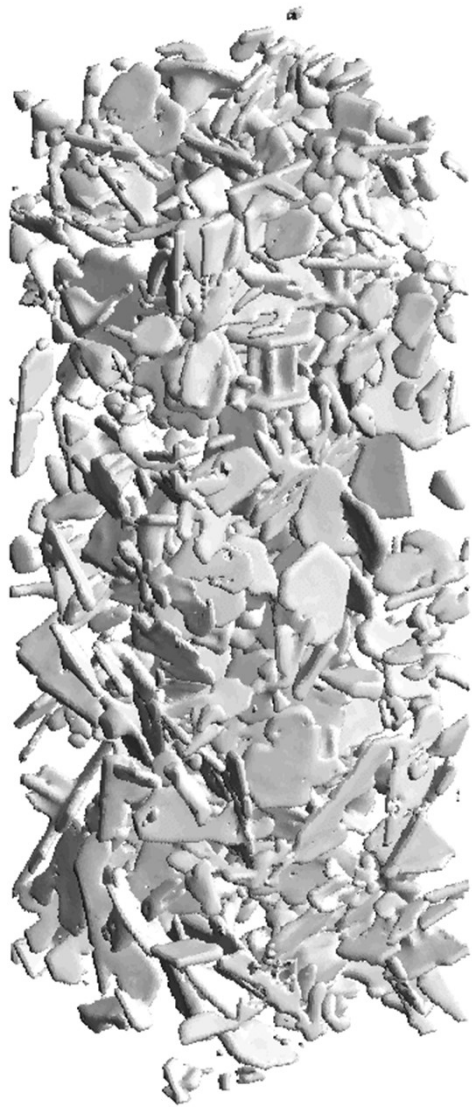


Conclusions:

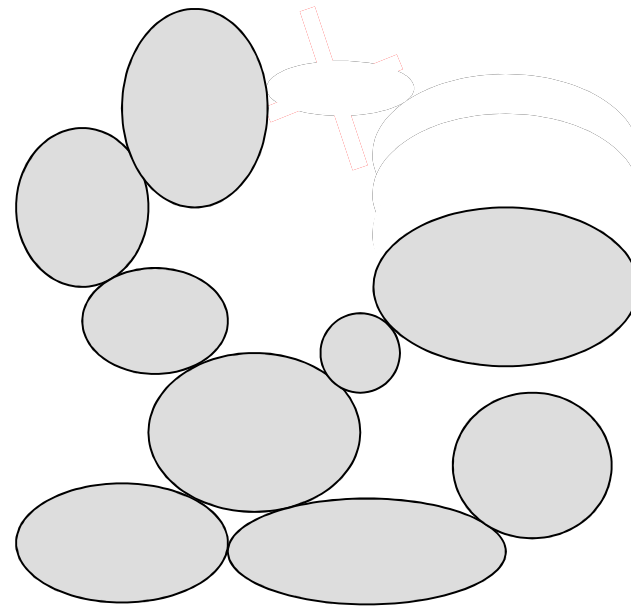
- Application of a 3D phase-field model developed for curvature flow with mass conservation
 - The model is applicable on tomographic images, and gives consistent results for the isothermal metamorphism of snow
 - It allowed calibrating the deposition coefficient α at -2°C , and this result has been confirmed by an independent time series
- ➔ Better understanding of the physical mechanisms involved

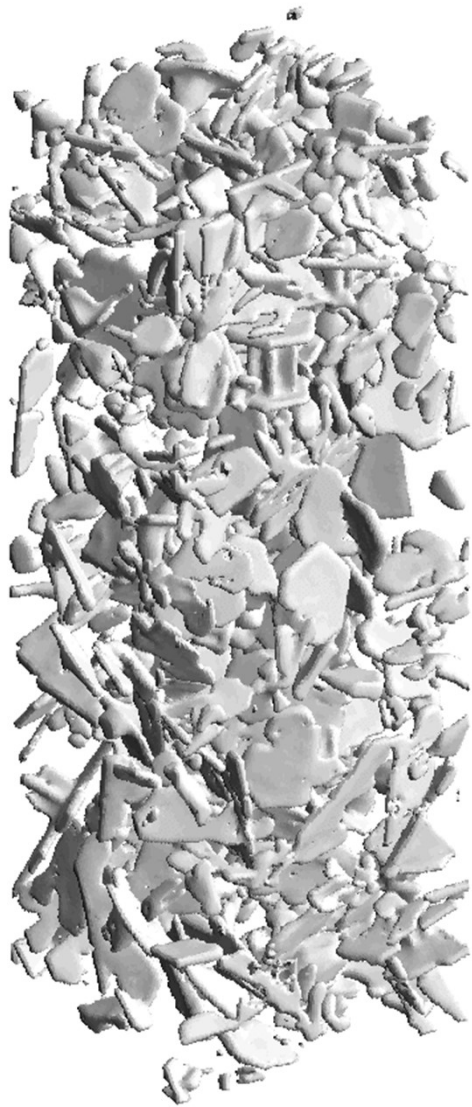
Perspectives:

- At the beginning of the simulation: elongated grains are disconnected by curvature effects. How modeling their mechanical rearrangement?
 - Extension to wet snow metamorphism (add another phase)
 - Extension to Temperature Gradient metamorphism (add crystal growth effects, thermal field computation...)
- ➔ Suggestions are welcome!



An attempt to simulate the grain
rearrangement





An attempt to simulate the grain
rearrangement

Densification caused
by the breaking of the necks

Evaporation-condensation → densification

Established principle in Material Sciences:
the evaporation-condensation mechanism
happens without any densification